Minnesota State High School Mathematics League

2019-20 Meet 1, Individual Event A

SOLUTIONS

NO CALCULATORS are allowed on this event.

$\boxed{\dfrac{5}{3}}$

1. Express $\dfrac{\frac{4}{3}+\frac{5}{4}}{\frac{3}{4}+\frac{4}{5}}$ as a quotient of two relatively prime integers.

$$\frac{\frac{4}{3}+\frac{5}{4}}{\frac{3}{4}+\frac{4}{5}}=\frac{\frac{16+15}{12}}{\frac{15+16}{20}}=\frac{20}{12}=\frac{5}{3}$$

$\boxed{45}$

2. Let b be a positive integer. For how many values of b is 21_b a two-digit number in base 10 ?

$10 \le 2b+1 \le 99 \to 4.5 \le b \le 49$.*and so b takes on values from 5 to 49, inclusive. There are* $49-5+1=45$ *values.*

$\boxed{\dfrac{60}{11}}$

3. Determine exactly the smallest positive rational number which when divided by $\dfrac{4}{11}$ or $\dfrac{3}{22}$ or $\dfrac{5}{33}$ always yields an integer?

The numerator must be the LCM of 4, 3. and 5 while the denominator must be the GCD of 11, 22, 33. Thus the number is $\dfrac{60}{11}$. $\left(\dfrac{60}{11}\div\dfrac{4}{11}=15,\ \dfrac{60}{11}\div\dfrac{3}{22}=40, \text{ and } \dfrac{60}{11}\div\dfrac{5}{33}=36.\right)$

$\boxed{5}$

4. Determine the number of ordered triples of digits $(\underline{A},\underline{B},\underline{C})$, such that $.\overline{\underline{A}\underline{B}} \div .\overline{\underline{C}\underline{A}} = 2$, that is, a decimal with a two digit repetend divided by a decimal with a two digit repetend equals 2.

Note that $.\overline{MN}=\dfrac{10M+N}{99}$. *Therefore,* $\dfrac{\frac{10A+B}{99}}{\frac{10C+A}{99}}=2 \to 10A+B=20C+2A \to 8A+B=20C$.

If $C=1$ *then* $A=2$ *and* $B=4$. *If* $C=2$, $A=5$ *and* $B=0$ *or* $A=4$ *and* $B=8$. *If* $C=3$, $A=7$ *and* $B=4$. *If* $C=4$, $A=9$ *and* $B=8$. *If* $C>5$, *then either* $A\ge 10$ *or* $B\ge 10$. *So there are 5 triples satisfying the problem.*