Minnesota State High School Mathematics League

2019-20 Meet 1, Individual Event B

SOLUTIONS

60°

1. In *Figure 1*, $ABCDEFGH$ is a cube. What is $m\angle EBD$? (Hint: The answer is not 90°.)

Draw $\overline{ED}$. Since each segment is the diagonal of a face of the cube, $\triangle EBD$ must be equilateral. Therefore, $m\angle EBD = 60^\circ$.

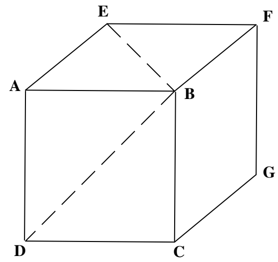

Figure 1

360°

2. In *Figure 2*, determine exactly the sum of the angles labelled 1 through 10.

$$5\left(180^\circ\right) - 3\left(180^\circ\right) = 2\left(180^\circ\right) = 360^\circ$$

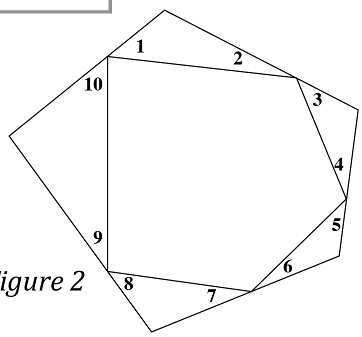

Figure 2

6

3. The interior angles of a convex polygon increase in the following linear progression: 100°, 108°, 116°, … . Determine the number of sides of the polygon.

The exterior angles of this polygon must also form a linear progression: 80°, 72°, 64°, … . *Since the exterior angles of any convex polygon add up to* 360°, *continue adding to the progression until the total reaches* 360°. $80^\circ + 72^\circ + 64^\circ + 56^\circ + 48^\circ + 40^\circ = 360^\circ$. *Therefore this must be a 6-sided polygon.*

40

4. The sides of right triangle ABC are a, $a+7d$, and $a+9d$ with *a* and *d* being integers. What is the smallest possible perimeter of $\triangle ABC$?

$a^2 + (a+7d)^2 = (a+9d)^2 \rightarrow a^2 + a^2 + 14ad + 49d^2 = a^2 + 18ad + 81d^2 \rightarrow a^2 - 4ad - 32d^2 = 0$. *This factors into* $(a+4d)(a-8d) = 0$. *So* $a = -4d$ *or* $a = 8d$. *The first solution is impossible but the second yields a triangle with sides* $8d$, $15d$, and $17d$. *If* $d = 1$, *the perimeter would be 40.*