Minnesota State High School Mathematics League

2019-20 Meet 1, Individual Event C

SOLUTIONS

NO CALCULATORS are allowed on this event.

$\boxed{1+\sqrt{3}}$

1. Determine exactly the value of $\sin\frac{\pi}{3}+\tan\frac{\pi}{4}+\cos\frac{\pi}{6}$.

$$\frac{\sqrt{3}}{2}+1+\frac{\sqrt{3}}{2}=1+\sqrt{3}$$

$\boxed{50}$ ***or*** $\boxed{50^\circ}$

2. Determine exactly the smallest positive integer n, such that $\sec\left(400^\circ\right)\cdot\sin\left(n^\circ\right)=1$.

$\sec\left(400^\circ\right)=\dfrac{1}{\cos\left(400^\circ\right)}=\dfrac{1}{\cos\left(40^\circ\right)}$. *Therefore,* $\sin\left(n^\circ\right)=\cos\left(40^\circ\right)$. *But* $\cos\left(40^\circ\right)=\sin\left(90^\circ-40^\circ\right)=\sin\left(50^\circ\right)$.

$\boxed{\sqrt{2}+1}$

3. $\triangle ABC$ has a right angle at B. If $BC=1$ and $\cos A=\frac{1}{3}$, determine exactly the perimeter of the triangle.

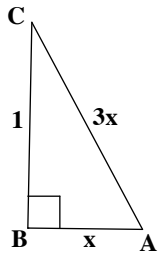

Figure 3.1

In Figure 3.1, $\cos A=\dfrac{AB}{AC}=\dfrac{x}{3x}$ *for some positive x. Therefore,* $1^2+x^2=9x^2$ *and* $x^2=\dfrac{1}{8}$ *and* $x=\dfrac{1}{\sqrt{8}}=\dfrac{\sqrt{2}}{4}$. *The perimeter is* $x+3x+1=4x+1=\sqrt{2}+1$.

$\boxed{-\frac{9}{4}}$ ***or*** $\boxed{-2.25}$ ***or*** $\boxed{-2\frac{1}{4}}$

4. In trapezoid $ABCD$, $\overline{AB}\parallel\overline{CD}$. If $AB=6$, $BC=8$, $CD=15$, and $AD=4$, determine exactly the value of $\cos A+2\cos B$.

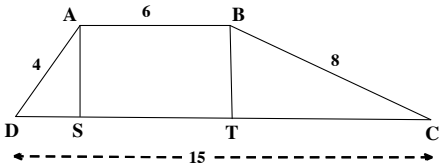

Figure 4.1

In Figure 4.1, draw altitudes $\overline{AS}$ *and* $\overline{BT}$. *Then* $\cos D=\dfrac{DS}{4}$ *and* $\cos C=\dfrac{CT}{8}$.

$DS+CT=4\cos D+8\cos C=15-6=9$. *Therefore,* $\cos D+2\cos C=\dfrac{9}{4}$. *Since* $\overline{AB}\parallel\overline{CD}$, $\cos A=-\cos D$ *and* $\cos B=-\cos C$. *So* $\cos A+2\cos B=-\dfrac{9}{4}$.