Minnesota State High School Mathematics League

2019-20 Meet 1, Individual Event D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points.
Place your answer to each question on the line provided. You have 12 minutes for this event.

_____   1.   Given $f(x) = 3x^5 + 5x^3 - 2x^2 + 82,$ determine exactly $f\left(f^{-1}(f(1))\right).$

_____   2. $f(x) = x^2 + bx + 12$. Determine for how many integer values of b, $f(x)$ has non-real

zeros.

$a =$ _____   3. $f(x) = ax^2$ with $a > 0$. An equilateral triangle with side length k is placed on the

parabola so that one of its vertices is on the vertex of the parabola and the other two
vertices are on $f(x)$. Write a formula for a, the leading coefficient of $f(x)$, in terms of

k. (Be sure to simplify.)

_____   4. $f(x) = -(x - r)(x - t)$ with $t > r$. A right triangle is placed on $f(x)$ such that two of its
vertices are $(r, 0)$ and $(t, 0)$ and its right angle vertex is on $f(x)$. Write a formula for the
area of this triangle in terms of r and t.

Name: _____   Team: _____