Minnesota State High School Mathematics League

2019-20 Meet 1, Individual Event D

SOLUTIONS

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1. Given $f(x)=3x^5+5x^3-2x^2+82$, determine exactly $f\left(f^{-1}\left(f(1)\right)\right)$.

$f^{-1}\left(f(x)\right)=x$. So $f\left(f^{-1}\left(f(1)\right)\right)=f(1)=3+5-2+82=88$.

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2. $f(x)=x^2+bx+12$. Determine for how many integer values of b, $f(x)$ has non-real zeros.

To have non-real zeros the determinant must be negative. So $b^2-4(1)(12)<0 \rightarrow b^2<48$. Since b is an integer, $-6 \le b \le 6$.

$a=\frac{2\sqrt{3}}{k}$

3. $f(x)=ax^2$ with $a>0$. An equilateral triangle with side length k is placed on the parabola so that one of its vertices is on the vertex of the parabola and the other two vertices are on $f(x)$. Write a formula for a, the leading coefficient of $f(x)$, in terms of k. (Be sure to simplify.)

The point $\left(\frac{k}{2},\frac{k\sqrt{3}}{2}\right)$ will be a point on this parabola. Therefore, $\frac{k\sqrt{3}}{2}=a\left(\frac{k}{2}\right)^2 \rightarrow$

$$a=\frac{4}{k^2}\cdot\frac{k\sqrt{3}}{2}=\frac{2\sqrt{3}}{k}.$$

$\frac{t-r}{2}$ **or** $\frac{1}{2}(t-r)$

4. $f(x)=-(x-r)(x-t)$ with $t>r$. A right triangle is placed on $f(x)$ such that two of its vertices are $(r,0)$ and $(t,0)$ and its right angle vertex is on $f(x)$. Write a formula for the area of this triangle in terms of r and t.

The third vertex must be $(n,f(n))$ for some value of n. Since the sides to the other two vertices are perpendicular at this point, $\frac{f(n)-0}{n-r}=-\frac{n-t}{f(n)-0} \rightarrow \left(f(n)\right)^2=-(n-r)(n-t) \rightarrow f(n)=1$. Therefore, the altitude of the triangle is 1 and the area is $\frac{1}{2}(t-r)(1)$.