Event C

1.

n	N
1	4
2	12
3	24
$\vdots$	

$N = an^2 + bn + c$ = sum of two consecutive squares minus one

$= (n+1)^2 + n^2 - 1$

$= n^2 + 2n + 1 + n^2 - 1$ (squares overlap)

$= 2n^2 + 2n + 0$

$= an^2 + bn + c$ $\therefore a+b+c = 2+2+0 = \boxed{4}$

2. License plate form lmn abc, where l, m, n are numbers, and a, b, c are
Let A = event lmn is a number palindrome
B = event abc is a letter palindrome

$P(A) = P(l=n) = \frac{1}{10}$ 10 digits

$P(B) = P(a=c) = \frac{1}{26}$ 26 letters

A and B are independent

$P(A \text{ or } B) = P(\text{only } A, \text{ not } B) + P(\text{not } A, \text{ only } B) + P(A \text{ and } B)$

$= \frac{1}{10} \cdot \frac{25}{26} + \frac{9}{10} \cdot \frac{1}{26} + \frac{1}{10} \cdot \frac{1}{26}$

$= \frac{25}{260} + \frac{9}{260} + \frac{1}{260} = \frac{35}{260}$

$= \frac{7}{52} = \frac{p}{q} \Rightarrow p+q = 7+52 = \boxed{59}$

Event D

1. $p(x) = 10x^2 - 29x + 21$. Roots are r_1, r_2
By Vieta, $r_1 r_2 = \frac{a_0}{a_n} = \frac{a_0}{a_2} = \frac{21}{10} = \frac{p}{q} \Rightarrow p+q = 21+10 = \boxed{31}$

2. 5 rows. Approach: Row-by-row, top, side, bottom

Top row SA $= 2(2\times6) + 2[2(1\times2) + 2(1\times6)] + 2(2\times2)$
$= 24 + 32 + 8 = 64$

2nd row SA $= 2(2\times2) + 2[2(1\times2) + 2(1\times6)] + 2(2\times2)$
$= 8 + 32 + 8 = 48$

3rd row SA = same as 2nd row = 48

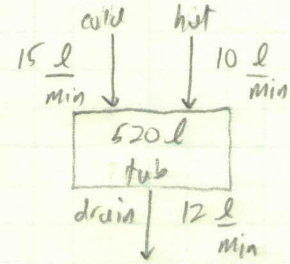
4th row SA = same as 2nd row = 48

5th row SA = same as 1st row = 64

Total SA = $2(64) + 3(48) = 128 + 144 = \boxed{272}$

Event A

1. Flux $= 15 + 10 - 12 = +13 \frac{l}{min}$ [flux is input − output]



$$\text{time} = \frac{520\ l}{13 \frac{l}{min}} = \boxed{40}\ min$$

2. $x^2 + 4x - 45 - 12\sqrt{5} = 0$ Solve. Hint:

$x^2 + 4x = 45 + 12\sqrt{5}$ must multiply to half of 12, or 6

$x^2 + 4x + 4 = 45 + 12\sqrt{5} + 4$

$(x+2)^2 = (3\sqrt{5} + 2)^2$ Note: $(6\sqrt{5} + 1)^2 = 180 + 12\sqrt{5} + 1 \neq 45 + 12\sqrt{5} + 4$

$x + 2 = \pm(3\sqrt{5} + 2)$

$x + 2 = 3\sqrt{5} + 2$ positive solution

$x = 3\sqrt{5}$

$= a\sqrt{b}$

$a + b = 3 + 5 = \boxed{8}$

Event B

1. Two similar, right hexagonal prisms have volumes 125 cm^3, 64 cm^3

scale factor $k = \frac{125}{64}$

Area of right prism is (area of base)(length) ⇒ ratio of bases is $\left(\frac{125}{64}\right)^{2/3}$

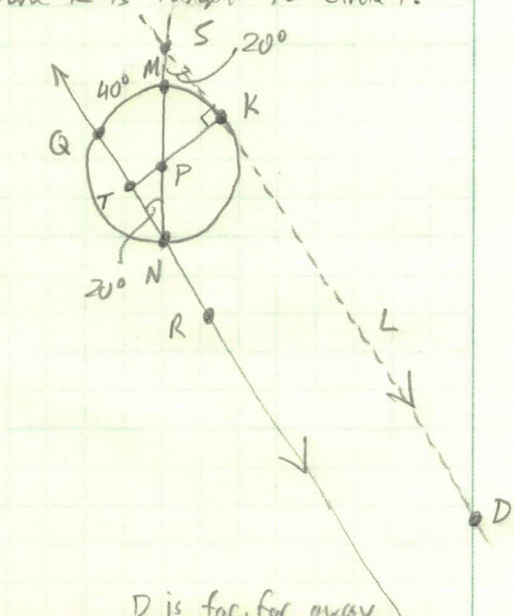
2 dimensions 1 dimension

⇒ ratio of bases is $\left(\frac{125}{64}\right)^{2/3} = \left[\left(\frac{125}{64}\right)^{1/3}\right]^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16} = \frac{p}{q}$

$\therefore p + q = 25 + 16 = \boxed{41}$

2. Case 1: Assume D far, far away. Draw $\overline{DK}$, where K is tangent to Circle P. Then $\overleftrightarrow{DN}$ and $\overleftrightarrow{DK}$ are nearly parallel

$m\,\widehat{QM} = 40° \Rightarrow m\angle QNM = \frac{40°}{2} = 20°$



Extend $\overline{MN}$ to point S on $\overleftrightarrow{DK}$

$m\angle PSK = m\,QNM = 20°$ (alternate interior angles)

Draw radius from K through P to T on $\overleftrightarrow{DN}$

$m\angle SKP = 90°$ (radius ⊥ tangent)

$m\angle SPK = 180° - m\angle PSK - m\angle SKP$

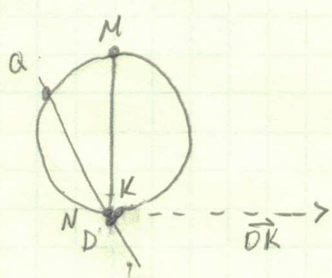
$= 180° - 20° - 90° = 70°$

$m\,\widehat{MK} = m\angle SPK = 70°$

$m\,\widehat{KN} = 180° - m\,\widehat{MK} = 180° - 70° = 110°$

D is far, far away such that $\overleftrightarrow{ND}$ and $\overleftrightarrow{KD}$ are nearly parallel

Case 2: Place D on (or very, very close to) N.
Then $D = K = N$ (or nearly)
$\overleftrightarrow{DN}$ approaches a tangent at N and $m\,\widehat{KN} = 0°$

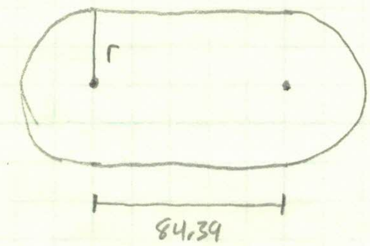


So interval is $(0°, 110°)$
(a, b)

$\therefore a + b = 0 + 110° = \boxed{110°}$

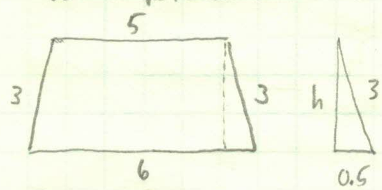
Team Event

1. r = radius of lane 8 runner
$= 36.5 + 7(1.22) + 0.2 = 36.5 + 8.54 + 0.2$
$= 45.24$ m
d = distance of lane 8 runner in one lap
$= 2\pi r + 2(84.39)$
$= 2\pi(45.24) + 2(84.39)$
$= 453.0313033$
$\cong \boxed{453}$ m



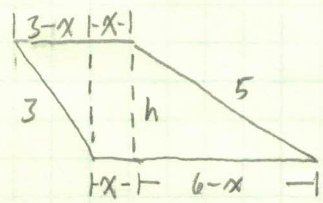
lane width = 1.22 m
lane 8 margin = 0.2 m

2. Two trapezoids with side lengths 3, 3, 5, 6



$h = \sqrt{3^2 - \left(\frac{1}{2}\right)^2} = \sqrt{9 - \frac{1}{4}} = \sqrt{\frac{35}{4}} = \frac{\sqrt{35}}{2}$

$A = \left(\frac{5+6}{2}\right)\frac{\sqrt{35}}{2} = \frac{11\sqrt{35}}{4} = \frac{a\sqrt{b}}{4} \Rightarrow a = 11, b = 35$



$h = \sqrt{3^2 - (3-x)^2} = \sqrt{5^2 - (6-x)^2}$
$9 - 9 + 6x - \cancel{x^2} = 25 - 36 + 12x - \cancel{x^2}$
$6x = -11 + 12x$
$11 = 6x \Rightarrow x = \frac{11}{6} \Rightarrow 3 - x = \frac{18-11}{6} = \frac{7}{6}$

$h = \sqrt{9 - \left(\frac{7}{6}\right)^2} = \sqrt{9 - \frac{49}{36}} = \sqrt{\frac{275}{36}} = \frac{5\sqrt{11}}{6}$

$A = \frac{3+6}{2} \cdot \frac{5\sqrt{11}}{6} = \frac{\cancel{9}^{3}}{2} \cdot \frac{5\sqrt{11}}{\cancel{6}_{2}} = \frac{15\sqrt{11}}{4} = \frac{c\sqrt{d}}{4} \Rightarrow c = 15, d = 11$

$\therefore a + b + c + d = 11 + 35 + 15 + 11 = \boxed{72}$

3. How many 0's in 1, 2, 3, …, 999, 1000?
Partition in a logical way (several ways are reasonable)

1-100:	11		11
101-109:	9	same for 201, 301, …, 801	$8 \cdot (9+11) = 8 \cdot 20 = \underline{160}$
110-200	11		
⋮			
901-909	9		9
910-1000	11+1		12

$11 + 160 + 9 + 12 = \boxed{192}$