<u>Determinants</u> (Meet 3 Event A topic)

A <u>matrix</u> is a rectangular array of numbers.

Examples

$$\begin{bmatrix}1&2\\3&4\end{bmatrix} \quad \begin{bmatrix}0&2&4\\1&3&5\\0&4&2\end{bmatrix} \quad \begin{bmatrix}3\end{bmatrix} \quad \begin{bmatrix}1&3&5\\6&4&2\end{bmatrix}$$

2×2   3×3   1×1   2×3

Matrices are usually represented as capital Latin letters: A, B, C, D, ...

rows × columns

Matrices are often used to solve systems of equations (n equations in n unknowns) in an efficient manner.

The <u>determinant</u> of a matrix is a single number that is calculated from a square matrix. (A <u>square matrix</u> has the same number of rows and columns.)

The determinant of a 1×1 matrix is simply the single entry in the matrix

Example: If $A = [3]$ then $\det A = |A| = 3$

The determinant of a 2×2 matrix is determined as follows

If $A = \begin{bmatrix}a&b\\c&d\end{bmatrix}$, where $a, b, c, d \in \mathbb{R}$, then $|A| = ad - bc$

Example: If $A = \begin{bmatrix}1&2\\3&4\end{bmatrix}$, then $|A| = 1\cdot4 - 2\cdot3 = 4 - 6 = \boxed{-2}$

$B = \begin{bmatrix}3&1\\2&5\end{bmatrix}$. Find $|B|$. $|B| = 3\cdot5 - 1\cdot2 = 15 - 2 = 13$ ✓

The determinant of a 3×3 matrix may be determined in (at least) 2 ways.

Method 1: Extending the diagonals

Example. Find $\det A = \begin{vmatrix}5&0&1\\-4&2&-7\\3&-6&8\end{vmatrix}$.

$$\begin{matrix}5&0&1&5&0\\-4&2&-7&-4&2\\3&-6&8&3&-6\end{matrix}$$

$-\ -\ -$

$+\ +\ +$

$$\det A = 5\cdot2\cdot8 + 0(-7)3 + 1(-4)(-6) - 3\cdot2\cdot1 - (-6)(-7)5 - 8(-4)0$$
$$= 80 + 0 + 24 - 6 - 210 + 0 = \boxed{-112}$$

Mr. Mac preference →

Method 2: Expansion of minors

+/− pattern: $\begin{bmatrix}+&-&+\\-&+&-\\+&-&+\end{bmatrix}$

Same example as above

$$\det A = 5\begin{vmatrix}2&-7\\-6&8\end{vmatrix} - 0\begin{vmatrix}-4&-7\\3&8\end{vmatrix} + 1\begin{vmatrix}-4&2\\3&-6\end{vmatrix}$$
$$= 5(16-42) - 0 + 1(24-6)$$
$$= 5(-26) + 18 = \boxed{-112}$$

Determinants (cont'd) (Meet 3 Event A topic)

Find the determinant

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 5 & -1 & 4 \\ 7 & 9 & -3 \end{bmatrix} = +0 \begin{vmatrix} -1 & 4 \\ 9 & -3 \end{vmatrix} - 2 \begin{vmatrix} 5 & 4 \\ 7 & -3 \end{vmatrix} + 3 \begin{vmatrix} 5 & -1 \\ 7 & 9 \end{vmatrix} =$$

$$= 0 \quad - 2(-15-28) + 3(45+7)$$

$$= -2(-43) + 3(52)$$

$$= 86 + 156$$

$$= \boxed{242}$$

$$B = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -6 \\ -7 & -8 & 9 \end{bmatrix} = -4 \begin{vmatrix} 2 & -3 \\ -8 & 9 \end{vmatrix} + 0 \begin{vmatrix} & \\ & \end{vmatrix} + 6 \begin{vmatrix} 1 & 2 \\ -7 & -8 \end{vmatrix}$$

$$= -4(18-24) + 6(-8+14)$$

$$= -4(-6) + 6(6)$$

$$= 24 + 36$$

$$= \boxed{60}$$

Application of system of equations (motivation)

Example: Given

$$\begin{aligned} x + 2y - 3z &= 2 \\ 4x \quad\;\; - 6z &= 4 \\ -7x - 8y + 9z &= 6 \end{aligned}$$

, we can rewrite this system of equations as

are 3×1 vectors

$$\begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & -6 \\ -7 & -8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

A vector is a matrix with either one column (a column vector) or one row (a row vector)

$$B \cdot \vec{x} = \vec{k}$$

An arrow over a (bolded) variable represents a vector

"goes away" — $B^{-1} B \cdot \vec{x} = B^{-1} \vec{k}$

$$\vec{x} = B^{-1} \vec{k}$$

I haven't told you how to calculate B^{-1} (B inverse)