Introduction

Matchstick puzzles are fun and have a long history. Matchstick magic tricks even appeared in the first book on recreational mathematics, *Problemes, Plaisans, et Delectables*, by Claude Gaspar Bachet, published in France in 1612. Wooden matchsticks work best, but paper matches, toothpicks, and same-length pencils can work too. You will need at most 24 "matches" to form these puzzles.

Problems

1. Remove six matches, leaving 10. [#1 in Martin Gardner, "Matches," *Mathematical Circus*, 1979; #852 in Arthur Hirschberg, *Can You Solve It?*, 1926]



2. The six matches are shown forming a map requiring three colors, assuming that no two regions sharing part of a border can have the same color. Rearrange the six to form a planar map requiring four colors. Confining the map to the plane rules out the three-dimensional solution of forming a tetrahedral skeleton. [#2 in Martin Gardner, "Matches," *Mathematical Circus*, 1979]



3. Rearrange the 12 matches to spell what matches are made of. [#3 in Martin Gardner, "Matches," *Mathematical Circus*, 1979]



4. Change the position of two matches to reduce the number of unit squares from five to four. "Loose ends"—matches not used as sides of unit squares—are not allowed. An amusing feature of this classic is that, even if someone solves it, you can set up the pattern again in mirror-reflected form or upside down (or both) and the solution will be as difficult as before. [#4 in Martin Gardner, "Matches," Mathematical Circus, 1979]



5. It is easy to see how to remove four matches and leave two equilateral triangles, but can you remove just two matches and leave two equilateral triangles? There must be no "loose ends." [#5 in Martin Gardner, "Matches," *Mathematical Circus*, 1979]



6. Move one match to produce a valid equation. Crossing the equality sign with a matcvh to make it a notequal sign is ruled out. [#6 in Martin Gardner, "Matches," *Mathematical Circus*, 1979]



7. Move one match to make a square. (The old joke solution of sliding the top match up a trifle to form a square hole at the center is not permitted; here the solution is a different kind of joke.) [#7 in Martin Gardner, "Matches," *Mathematical Circus*, 1979]



Arrange six matchsticks as indicated. Add five matchsticks and make nine. [#851 in Arthur Hirschberg, Can You Solve It?, 1926]

8.



9. Again, arrange the fifteen matches in the diagram. Remove six again and leave a hundred. [#853 in Arthur Hirschberg, Can You Solve It?, 1926]



10. Arrange sixteen matches as in the diagram. Rearrange three matches and leave four squares. [#854 in Arthur Hirschberg, *Can You Solve It?*, 1926]



11. Remove five matches and leave three squares. [#855 in Arthur Hirschberg, Can You Solve It?, 1926]



12. Form eight squares with 22 matches, as shown in the diagram. Remove six matches and leave four squares. [#856 in Arthur Hirschberg, *Can You Solve It?*, 1926]



13. With twenty matches form seven squares, as arranged in the diagram. Rearrange three matches and leave five squares. [#857 in Arthur Hirschberg, *Can You Solve It?*, 1926]



14. Arrange 24 matches to form 9 squares, as in the diagram. Remove 8 matches and leave two squares. [#858 in Arthur Hirschberg, *Can You Solve It?*, 1926]



15. Arrange twelve matches to form four squares, as in the diagram. Rearrange four matches and form three squares, each the same size as the original ones. [#859 in Arthur Hirschberg, *Can You Solve It*?, 1926]



- 16. With six matches of equal length, construct four equilateral triangles. [#860 in Arthur Hirschberg, Can You Solve It?, 1926]
- 17. With nine matches form five equilateral triangles. [#861 in Arthur Hirschberg, Can You Solve It?, 1926]
- 18. With nine matches, form three equal squares and two congruent triangles. [#862 in Arthur Hirschberg, Can You Solve *It?*, 1926]
- 19. With six matchsticks form a figure as in the diagram. Then by moving only two matches, and adding one more, form two diamonds. [#863 in Arthur Hirschberg, *Can You Solve It?*, 1926]
- 20. Arrange 24 matches to form six squares and twenty triangles. [#864 in Arthur Hirschberg, Can You Solve It?, 1926]

21. Assuming that a match is a unit of length, it is possible to place 12 matches on a plane in various ways to form polygons with integral areas. The illustration shows two such polygons: a square with an area of nine square units, and a cross with an area of five. The problem is this: Use all 12 matches (the entire length of each match must be used) to form in similar fashion the perimeter of a polygon with an area of exactly four square units. [#14, "The 12 Matches," in Martin Gardner, *My Best Mathematical & Logic Puzzles*, 1994]



22. Arrange four matches on the table as shown in the left-hand figure. They represent a martini glass. A match head goes inside to indicate the onion of a Gibson cocktail. The puzzle is to move just *two* matches so that the glass is reformed, but the onion—which must stay where it is—winds up *outside* the glass. At the finish, the glass may be turned to the left or right, or even be upside down, but it must be exactly the same shape as before. The upper right figure is not a solution because the onion is still inside. The lower right figure doesn't work because *three* matches have been moved. [#55, "Out with the Onion," in Martin Gardner, *My Best Mathematical & Logic Puzzles*, 1994]



23. Arrange 13 matches to make a dog that faces to the left as in the diagram. By lowering the dog's tail to the top dotted line, then moving the bottom match of the dog's head to the other dotted line, you have changed the picture so that the dog is looking the opposite way. Unfortunately, this leaves the dog's tail (now on the left) slanted down instead of up. Can you move just two matches to make the dog face to the right, but with his tail pointing upward as before? [#65, "Reverse the Dog," in Martin Gardner, *My Best Mathematical & Logic Puzzles*, 1994]



Aftermath

- 1. Devise methods whereby you can determine the number of matches in each of the diagrams here at a glance.
- 2. Devise your own matchstick puzzles. Trade them with another person and solve each other's puzzles.

References

Garner, Martin. (1979). *Mathematical Circus*. Random House.

Garner, Martin. (1994). *My Best Mathematical and Logic Puzzles*. Dover Publications.

Hirschberg, Arthur. (1926). Can You Solve It? A Book of Puzzles and Problems. Thomas Y. Crowell Company.

Author's Diagram-Formation Area

These are not puzzles. Rather, this is the area the author used to construct the diagrams in the paper. To support consistency and future paper modification, these original images are retained.

Author instructions: Use the individual matchstick images below to form matchstick diagrams. Then capture the collection as a single image and paste within the document.







