Math Team Notes
Topic 2A: Linear Equations in One Unknown

Subtopics

Topic 2A, Linear equations in one unknown, includes the following subtopics.

<u>**2A Algebra 1: Linear equations in one unknown**</u>
2A1 Solving numeric equations (perhaps involving a second-degree term that drops out)
2A2 Solving literal equations
2A3 Story problems leading to linear equations in one variable
2A4 Linear inequalities
2A5 Absolute value equations and inequalities

Notes

- **Definition** A ***numeric equation*** is simply what we would call an equation. For example, $2x - 8 = 6$ is a (numeric) equation.
- **Definition** To ***solve*** an equation is to isolate one variable on one side of the equation. This is accomplished by applying the same operation to each side of the equation one or more times (and simplifying as necessary) to isolate one variable on one side of the equation. For example, the equation $2x - 8 = 6$ may be solved for x by first adding 8 to each side (obtaining $2x = 6 - 8$), simplifying (obtaining $2x = -2$) and then dividing each side by 2 (obtaining $x = -1$).
- **Definition** A ***second-degree term*** is a term of a polynomial in which the exponent of the variable is 2. For example, in the polynomial $3x^2 + 4x + 5$, the second-degree term is $3x^2$. (Also, the *first-degree term* is $4x$ and the *zeroth-degree term* is 5, because $5 = 5x^0 = 5 \cdot 1 = 5$).
- The most efficient method in solving an equation is typically to apply inverse operations in the reverse sequence of the order of operations in evaluating an expression. For example, in solving the equation $2x - 8 = 6$, one first *adds* 8 to each side and then *divides* each side by 2.
- The order of operations in *evaluating* an expression may be expressed as
    1. Grouping: parentheses (), brackets [], braces {} from the inside-out; numerators and denominators; radicands (inside square roots, cube roots, etc.)
    2. Exponents and roots
    3. Multiplication and division, left to right
    4. Addition and subtraction, left to right
- **Definition** A ***literal equation*** is an equation with two or more variables. For example, the equation $2x + y = 6$ is a literal equation, which may be solved for either variable y (as $y = -2x + 6$) or x (as $x = \frac{1}{2}y + 3$).
- **Definition** A ***story problem*** (also known as a ***word problem*** or sometimes a ***real-world application***) is a problem in which the person solving the problem must first determine the equation to be solved from the problem description, which is typically expressed using words and numbers.
- **Definition** The ***absolute value of a real number*** x is denoted $|x|$ and defined as the "unsigned" portion of x. For example, the absolute value of 3 is $|3| = 3$ and the absolute value of -3 is $|-3| = 3$. The absolute value of a real number is always positive or zero.
- **Definition** An ***absolute value equation*** is an equation with an absolute value expression. For example, the equation $2|x - 7| - 6 = 14$ is an absolute value equation.
- To solve an absolute value equation, first isolate the absolute value expression. Second, use the definition of absolute value to express the absolute value equation as two equations, each without an absolute value. Third, solve each of the two equations. The solution is the set of values that satisfy at least one of the equations. For example, to solve the equation $2|x| - 6 = 14$, first isolate the absolute value expression (obtaining $|x - 7| = 10$). Then express this

equation as two equations (obtaining $x - 7 = 10$ and $x - 7 = -10$). Then solve each equation (obtaining $x = 17$ and $x = -3$). The solution may be expressed in multiple ways, such as (1) $x = -3$ or $x = 17$, (2) $x = -3$ or 17, (3) $x = -3, 17$, or (4) $x \in \{-3, 17\}$.

- Alternately, first isolate the absolute value expression. Second, graph each side of the equation. Third, determine the intersection(s). For example, to solve the equation $2|x| - 6 = 14$, first isolate the absolute value expression (obtaining $|x - 7| = 10$). Then graph $y = |x - 7|$ and $y = 10$. Then note that the intersections are when $= -3, 17$.
- To solve an absolute value inequality with more than one absolute value expressions, set each expression to zero and solve for x. Partition the real numbers into regions separated by these x values. Solve the inequality one case (region) at a time, negating the expression within each absolute value as necessary. Summarize the result.

Problems

For the following problems, assume a calculator is not allowed unless stated.

<u>Problem #1 (“quickie”; 1 point)</u>

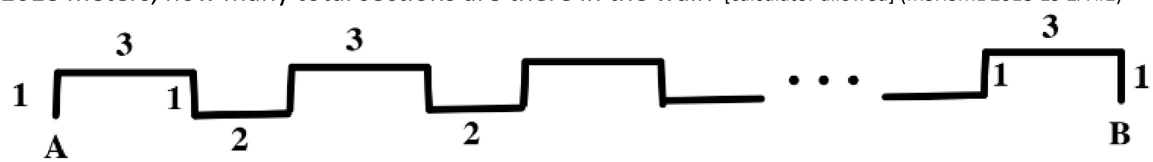
Goal: Know this topic so well that you can solve a Minnesota State High School Mathematics League (MSHSML) problem #1 in less than one minute.

1. Find the sum of the two solutions to this absolute value equation: $|2x + 1| = 9$. [calculator allowed] (MSHSML 2019-20 2A #1)
2. Sal earns $30.00 for a day’s work but also receives a commission of 5% on all the merchandise she sells. If she earned $120.00 yesterday, how much merchandise did she sell? [calculator allowed] (MSHSML 2018-19 2A #1)

<u>Problem #2 (“textbook”; 2 points)</u>

Goal: Know this topic so well that you can solve an MSHSML problem #2 in less than two minutes.

1. I have nickels, dimes, and quarters in my pocket. The total of this change is $3.90. I have twice as many nickels as dimes and half as many quarters as one of the other coins. How many dimes do I have? [calculator allowed] (MSHSML 2019-20 2A #2)
2. The figure below is a portion of a highway wall as seen from above. The vertical sections are each 1 meter wide, the upper horizontal sections are each 3 meters wide, and the lower horizontal sections are 2 meters wide. If the length of the wall, i.e., the straight-line distance from A to B is 2018 meters, how many total sections are there in the wall? [calculator allowed] (MSHSML 2018-19 2A #2)



<u>Problem #3 (“textbook with a twist”; 2 points)</u>

Goal: Know this topic so well that you can solve an MSHSML problem #3 in less than three minutes.

1. I invested $1000.00 and earned 5% interest the first year. I then removed $200.00 and the second year the account lost 4%. I added the $200.00 back into the account and the third year the account earned 4%. Determine how much more or less I would have made if I had left the $200.00 in the account. Circle “more” or “less” (printed in the answer area) in the answer. [calculator allowed] (MSHSML 2019-20 2A #3)
2. Find the value of m such that $\frac{x+m}{x+3} + \frac{x-m}{x+2} = \frac{2x+1}{x+3}$, for all x except $x = -3$ and $x = -2$. [calculator allowed] (MSHSML 2018-19 2A #3)

Problem #4 (“challenge”; 2 points)

Goal: Know this topic so well that you can solve an MSHSML problem #4 in less than six minutes.

1. Find the interval for x in which $|2x+4| \leq |2x-4| + 4$. [calculator allowed] (MSHSML 2019-20 2A #4)
2. Samantha left home at 8:00 am and, hiking at 3 mph, arrived at the chalet on top of a mountain at 4:00 pm. The next morning she left at 9:00 am and, hiking at 5 mph, returned home. It turns out that she passed one spot on the trail at exactly the same time on both days. At what time did that occur? Write your answe3r as hour: minute: second and use “am” or “pm”. For example: “10:12:30 am” is twelve and a half minutes after ten in the morning. [calculator allowed] (MSHSML 2018-19 2A #4)

If you are able to solve MSHSML problem #s 1, 2, and 3, in less than 1, 2, and 3 minutes, respectively, you will have at least 6 minutes (assuming a 12-minute, 4-question exam) to solve problem #4 (“challenge problem”; 2 points). Problem #4 tends to be more varied in nature than problems #1-3 and may require a broader knowledge of other mathematical areas (geometry, for example). For more MSHSML Meet 2 Event A problems, see past exams, which date back to 1980-81.