MNJHML Meet 1 Answers

1. 100
2. 23
3. 25
4. $(6-4)\cdot(7+4)-3=19$
5. The following are all possibilities with a single set of parentheses:
   i. $(2+3)\times 5-7=18$
   ii. $(2+3\times 5)-7=10$
   iii. $(2+3\times 5-7)=10$
   iv. $2+(3\times 5)-7=10$
   v. $2+(3\times 5-7)=10$
   vi. $2+3\times(5-7)=-4$
6. $=\frac{19}{30}$
7. 25
8. 225 cabins
9. 2.95
10. $\frac{16}{9}$
11. 28
12. 11
13. $108\sqrt{2}$
14. $n=26$
15. $x=6$
16. 720 billion positions during a one-hour match.
17. 12 minutes
18. ⅓ of a mile.
19. The girls will meet 1.5 miles from Hafsa's house.
20. 45°.
21. 40°
22. *ADB* = 115°.
23. *BCA* = 60°.
24. 900 four-digit numbers.
25. 18 combinations.
26. 360 different security codes
27. 529,000 unique license plates.
28. 1,440 permutations

MNJHML Meet 1 Complete Solutions

Order of Operations, including Invented Operators

1. $3 \cdot 27 - 128 \div 2 + 19 + 2 \cdot 32 = (3 \cdot 27) - (128 \div 2) + 19 + (2 \cdot 32)$
   $= 81 - 64 + 19 + 64$
   $= 81 + 19$
   $= 100$

2. $7 \ominus (1 \ominus 5) = 7 \ominus (3 \cdot 1 - 5)$
   $= 7 \ominus (-2)$
   $= 3 \cdot 7 - (-2)$
   $= 21 + 2$
   $= 23$

3. $(8 \lozenge 1) + (3 \lozenge 4) - (7 \lozenge 4) = (8^2 + 1^2) + (3^2 + 4^2) - (7^2 + 4^2)$
   $= (64 + 1) + (9 + 16) - (49 + 16)$
   $= 64 + 1 + 9 + 16 - 49 - 16$
   $= 64 + 1 + 9 - 49$
   $= 74 - 49$
   $= 25$

4. The following are all possibilities with a single set of parentheses:
   $(2 + 3) \times 5 - 7 = 18$
   $(2 + 3 \times 5) - 7 = 10$
   $(2 + 3 \times 5 - 7) = 10$
   $2 + (3 \times 5) - 7 = 10$
   $2 + (3 \times 5 - 7) = 10$
   $2 + 3 \times (5 - 7) = -4$

   We see that only the values 18, 10, and -4 are possible, so the number of different values which can be obtained in this manner is

5. This isn't possible with a single single set of parentheses, however it can be done using two:

   $(6 - 4) \cdot (7 + 4) - 3 = 19$

Rational Number Concepts including Decimals and Fractions

6. We use a common denominator of 30:

$$\frac{3}{10}+\frac{2}{5}-\frac{1}{15}=\frac{3}{10}\cdot\frac{3}{3}+\frac{2}{5}\cdot\frac{6}{6}-\frac{1}{15}\cdot\frac{2}{2}$$
$$=\frac{9}{30}+\frac{12}{30}-\frac{2}{30}$$
$$=\frac{9+12-2}{30}$$
$$=\frac{19}{30}$$

7. Multiply top and bottom by an appropriate power of 10 to remove decimals:

$$\frac{4}{0.2}+\frac{0.6}{0.12}=\frac{4}{0.2}\cdot\frac{10}{10}+\frac{0.6}{0.12}\cdot\frac{100}{100}$$
$$=\frac{40}{2}+\frac{60}{12}$$
$$=20+5$$
$$=25$$

Alternately each fraction can be evaluated separately using long division (taking care to handle decimals correctly).

8. Four acres of forest contains 600 trees (4×150); the number of cabins that can be built from 600 trees is $600 \div 2\tfrac{2}{3}$.

Now, $600 \div 2\frac{2}{3} = 600 \div \frac{8}{3}$
$$= 600 \times \frac{3}{8}$$
$$=\frac{1800}{8}$$
$$= 225$$

So 225 cabins could be built using the wood from a four acre forest.

9. One method is to convert fractions to those whose denominators are powers of 10:

$$2+\frac{3}{5}+\frac{7}{20}=2+\frac{3}{5}\cdot\frac{2}{2}+\frac{7}{20}\cdot\frac{5}{5}$$

$$= 2 + \frac{6}{10} + \frac{35}{100}$$
$$= 2 + 0.6 + 0.35$$
$$= 2.95$$

10. We work from the innermost fraction out; if this isn't clear note that this expression can be written as follows to make this clearer:

$$1 + \frac{2}{2+\frac{2}{3+\frac{2}{4}}} = 1 + 2 \div (2 + 2 \div (3 + 2 \div 4)))$$

Then

$$1 + \frac{2}{2+\frac{2}{3+\frac{2}{4}}} = 1 + \frac{2}{2+\frac{2}{3+\frac{1}{2}}}$$
$$= 1 + \frac{2}{2+\frac{2}{(\frac{7}{2})}}$$
$$= 1 + \frac{2}{2+2\times(\frac{2}{7})}$$
$$= 1 + \frac{2}{2+\frac{4}{7}}$$
$$= 1 + \frac{2}{(\frac{18}{7})}$$
$$= 1 + 2 \times \frac{7}{18}$$
$$= 1 + \frac{7}{9}$$
$$= \frac{16}{9}$$

Alternately each fraction can be evaluated separately using long division.

Positive Exponents and Roots

11. $2^6 - 6^2 = 64 - 36$
$= 28$

12. Note that $2 \cdot 5 = 10$; we can use this product to our advantage:

$7^2 \cdot 2^{2017} \cdot 5^{2018} = 49 \cdot 2^{2017} \cdot 5^{2017} \cdot 5$

$= 49 \cdot 10^{2017} \cdot 5$

$= 49 \cdot 5 \cdot 10^{2017}$

$= 245,$ followed by 2017 zeros.

Therefore the sum of the digits is $2 + 4 + 5 = 11$

13. $\sqrt{3^5 \cdot 16 \cdot 6} = \sqrt{3^4 \cdot 3 \cdot 2^4 \cdot 2 \cdot 3}$

$= \sqrt{3^4 \cdot 3 \cdot 2^4 \cdot 2 \cdot 3}$

$= \sqrt{3^4} \cdot \sqrt{2^4} \cdot \sqrt{3 \cdot 2 \cdot 3}$

$= 3^2 \cdot 2^2\sqrt{2 \cdot 3^2}$

$= 9 \cdot 4 \cdot 3\sqrt{2}$

$= 108\sqrt{2}$

14. First, rewrite the powers with 3 as the base:

$27^7 \cdot 3^{15} \cdot 81^4 = \left(3^3\right)^7 \cdot 3^{15} \cdot \left(3^4\right)^4$

$= 3^{21} \cdot 3^{15} \cdot 3^{16}$

$= 3^{21+15+16}$

$= 3^{52}$

now rewrite this with a base of 9:

$= \left(3^2\right)^{26}$

$= 9^{26}$

Therefore, $n = 26$

15. $(6^3)^{11} = x \cdot (x^4)^8$

$6^{33} = x \cdot x^{32} = x^{33}$, so $x = 6$

Speed and Other Rates (Distance = Rate × Time)

16. Since there are 60 seconds in a minute, Deep Blue would evaluate 60×200 = 12,000 million positions per minute, which is 12 billion positions per minute.

17. With 60 minutes in an hour, it would evaluate 12×60 = 720 billion positions during a one-hour match.

    At a steady ½ meter per second, the tortoise will take 400 ÷ ½ = 800 seconds to complete the 400m race (since time = distance ÷ rate).

    Similarly if the hare hadn't stopped it would have taken 400 ÷ 5 = 80 seconds to complete the race.

    This means that the hare must have been asleep for exactly 800 - 80 = 720 seconds, which is 12 minutes (since 720 ÷ 60 = 12).

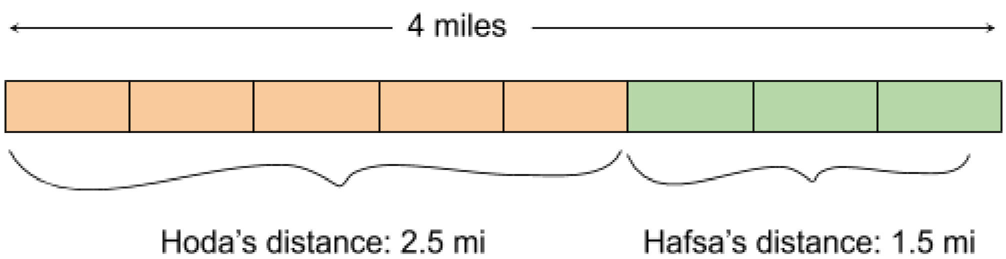
18. Into the current, the rower will move at a rate of 2 - 1 = 1 mile per hour with respect to the shore. In 40 minutes she will have traveled ⅔ mile (since 40 minutes is ⅔ of an hour)

    Rowing with the current, she will move at a rate of 2 + 1 = 3 miles per hour with respect to the shore. In 20 minutes she will have traveled 1 mile.

    Thus she will end up downstream from where she launched by 1 - ⅔ = ⅓ of a mile.

19. Since Hafsa runs at ⅗ the rate that Hoda runs, Hafsa will cover ⅗ the distance in the same amount of time; in other words for every 5 units Hoda runs, Hafsa will run 3 units.

    If they leave at the same time, they will each have run for the same amount of time when they meet, and the distances will sum to 4 miles. This means the 4 miles can be divided into *eight* equal half-mile segments, as shown. Therefore the girls will meet 1.5 miles from Hafsa's house.

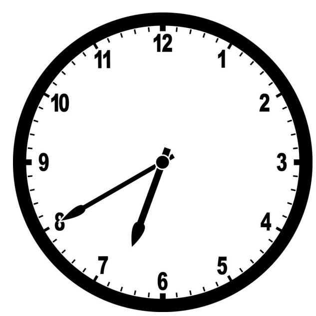


Finding Angle Measures

20. Since a straight line is 180°, the unmarked angle in the triangle is 180° - 107° = 73°.

    Since the angles in a triangle also add to 180°, $x = 180°$ - (62° + 73°) = 45°.

21. Since there are 12 hours in a standard clock, the hour hand sweeps over $\frac{360}{12} = 30$ degrees every hour. 6:40 is ⅔ of an hour past 6 o'clock, so at 6:40 the hour hand has moved ⅔ × 30 = 20° past the

    Similarly the minute hand sweeps 30 degrees every 5 minutes, and it pointed to the 6 at 6:30, so at 6:40 it will have moved 2 × 30 = 60° past the

    As a result, the acute angle between the hands of the clock will be 60° - 20° = 40° at 6:40.

22. The angles in triangle ABC add to 180°, so 50° plus twice angle x plus twice angle y is 180°. This means that twice angle x plus twice angle y is 130°, so angle x plus angle y must be 65°.

    Now the angles in triangle ABD also add to 180°, in other words angle x plus angle y plus angle *ADB* is 180°. But we know that angle x plus angle y adds to 65°, so angle *ADB* must be 180° - 65° = 115°.

23. Opposite angles in a parallelogram are equal, so angle *ABC* = 47°.

Then since the angles in triangle ABC add to 180^o, angle *BCA* = $180^o - (47^o + 73^o)$ which is 60^o.

Multiplication Principle and Counting with Permutations

24. There are nine possibilities for the thousands digit of our four-digit integer (1 - 9), since it can't start with zero.

    For each of these there is one possibility for the hundreds digit (2).

    For each of these there are 10 possibilities for the tens digit (0 - 9)

    For each of these there are 10 possibilities for the units digits (0 - 9).

    Thus by the multiplication principle, overall there are $9 \times 1 \times 10 \times 10 = 900$ such four-digit numbers.

25. There are three drink possibilities (milk, juice, or water). For each of these there are three entree possibilities (sandwich, wrap, or pizza). For each of these there are two vegetable options (carrot sticks or a salad).

    As a result by the multiplication principle there are $3 \times 3 \times 2 = 18$ different lunch combinations overall.

26. *Solution 1*:

    There are six options for the first digit of the security code (1-6).

    For each of these there are five options for the second digit (it can't equal the first digit)

    For each of these there are four options for the third digit (it can't equal either of the first two digits).

    For each of these there are three options for the fourth digit (it can't equal either of the first three digits).

As a result overall there are 6×5×4×3 = 360 different security codes satisfying these criteria.

Solution 2:

We recognize that a security code can be thought of as a permutation of four digits from the set {1, 2, 3, 4, 5, 6} which can be done in $P(6,4)$ ways.

But $P(6,\ 4) = \frac{6!}{(6-4)!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2\cdot1} = 360$ as before.

27. Any of 23 letters may be chosen for each of the first two digits, and any of 10 digits may be chosen for the each of last three, giving a total of 23×23×10×10×10 = 529,000 unique license plates.

28. For each of the 6 total two-point problems to select for problem 1, there are 5 options for problem 2, and 4 for problem 3, so there are 6×5×4 ways to select the first three. For each of those permutations, there are 4 ways to select problem 4, and 3 remaining ways to select problem 5. In total, there are (6×5×4)(4×3)=1,440 permutations