SMacLennan
ERA Math Team
Forum AB
Tu 20 Oct 2020

Math Team
Meet 1 Events A and B Problems 3 2018-20 Practice

Event A
Problem #3 ("textbook with a twist"; 2 points)
Try to solve each problem within three minutes.

we know $0.\overline{7} = \frac{7}{9}$

$0.\overline{76} = \frac{76}{99}$

3. Express $0.20\overline{18}$ as a quotient of two relatively prime integers. (MSHSML ~~2019-20~~ 2018-19 1A #3)

$x = 0.20\overline{18}$

[1] $100x = 20.\overline{18}$

[2] $10{,}000x = 2018.\overline{18}$

[2]-[1]: $9900x = 1998$ (2018-20)

$x = \frac{1998}{9900} = \frac{222}{1100} = \boxed{\frac{111}{550}}$ (3·37; $2 \cdot 5^2 \cdot 11$)

3. Express $0.20\overline{19}$ as a quotient of two relatively prime integers. (MSHSML 2018-19 1A #3) No!

$x = 0.20\overline{19}$

$100x = 20.\overline{19} = 20 + \frac{19}{99}$

$x = \frac{20 + \frac{19}{99}}{100}$

$x = \frac{20 \cdot 99 + 19}{\frac{99}{100}} = \frac{1980 + 19}{99} = \frac{1999}{99} \cdot \frac{1}{100} = \boxed{\frac{1999}{9900}}$

899
19

$2^2 \cdot 3^2 \cdot 5^2 \cdot 11$

× × ×

3. What is the base b for which $\underline{6}\,\underline{8}_b$ is 25% larger than $\underline{5}\,\underline{3}_b$? (Note that the percent is given in base 10.) (MSHSML 2017-18 1A #3)

we know $53_{10} = 5 \times 10^1 + 3 \times 1$

$68_b = 6 \cdot b^1 + 8 \cdot b^0 = 6b + 8$

$53_b = 5b + 3$

$6b + 8 = \frac{5}{4}(5b + 3)$

$24b + 32 = 25b + 15$

$\boxed{17} = b$

3. If 48 and x have a lowest common multiple of 2640 and a greatest common factor of 12, determine the minimum possible value of x. (MSHSML 2016-17 1A #3)

$48 = 2^4 \cdot 3$

$2640 = 2 \cdot 5 \cdot 264 = 2^4 \cdot 5 \cdot 33 = 2^4 \cdot 3 \cdot 5 \cdot 11$

$GCF(48, x) = 12 \Rightarrow x$ is a multiple of $12 \Rightarrow$ it has factors of $2^2 \cdot 3$

$LCM(48, x) = 2640 \Rightarrow x$ have factors of 5 and 11

Minimum $x = 2^2 \cdot 3 \cdot 5 \cdot 11 = 60 \cdot 11 = \boxed{660}$