

Math Team
Meet 1 Events C and D Problems #1-2 Practice 2016-18

Event C

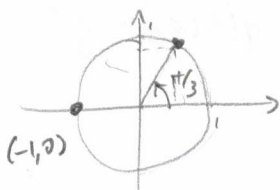
Problem #1 ("Quickie"; 1 point)

Try to solve each problem within one minute.

$2\sqrt{3}+2$

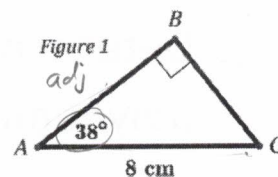
1. Determine exactly the value of $\sin \frac{\pi}{3} - \cos 3\pi$. (MSHSML 2015-16)

1C #1)



$$= \frac{\sqrt{3}}{2} - (-1) = \frac{\sqrt{3}}{2} + \frac{2}{2} = \frac{\sqrt{3}+2}{2}$$

1. Figure 1 shows $\triangle ABC$ with $m\angle A = 38^\circ$ and $AC = 8$ cm. Calculate the length of \overline{AB} . [calculator allowed] (MSHSML 2014-15 1C #1)



$$\cos 38^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{AB}{8} \Rightarrow AB = 8 \cos 38^\circ \approx \boxed{6.304}$$

Problem #2 ("Textbook"; 2 points)

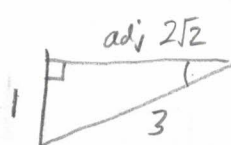
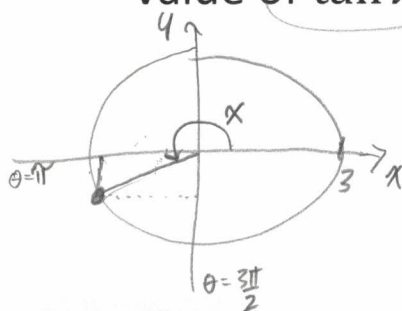
Try to solve each problem within two minutes.

2. If $\tan A = -\frac{\sqrt{39}}{5}$ and $\cos A = \frac{5}{8}$, determine exactly the value of $1 + \sin^2 A$. (MSHSML 2015-16 1C #2)

$$\tan A = \frac{\sin A}{\cos A} \Rightarrow \sin A = \tan A \cdot \cos A = -\frac{\sqrt{39}}{5} \cdot \frac{5}{8} = -\frac{\sqrt{39}}{8}$$

$$1 + \sin^2 A = 1 + \left(-\frac{\sqrt{39}}{8}\right)^2 = 1 + \frac{39}{64} = \frac{64}{64} + \frac{39}{64} = \boxed{\frac{103}{64}}$$

2. If $\sin x = -\frac{1}{3}$ and $\pi < x < \frac{3\pi}{2}$, determine exactly the value of $\tan x$. [calculator allowed] (MSHSML 2014-15 1C #2)



$$1^2 + \text{adj}^2 = 3^2$$

$$\text{adj}^2 = 9 - 1 = 8$$

$$\text{adj} = \sqrt{8} = 2\sqrt{2}$$

$$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{4}}$$

Event D

Problem #1 ("Quickie"; 1 point)

Try to solve each problem within one minute.

1. Let $f(x) = x + 3$ and $g(x) = x^2$. Determine exactly the value(s) of x for which $g(f(x)) = 0$. (MSHSML 2015-16 1D #1)

$$g(f(x)) = g(x+3) = (x+3)^2 = 0$$

$$g(f(x)) = (g \circ f)(x)$$

$$\therefore \boxed{x = -3}$$

1. Determine exactly the sum of the roots of the cubic polynomial $2x^3 - 9x^2 + 14x - 6$. (MSHSML 2014-15 1D #1)

Vieta's Formulas

$$\text{sum of roots} = -\frac{-9}{2} = \boxed{\frac{9}{2}}$$

Problem #2 ("Textbook"; 2 points)

Try to solve each problem within two minutes.

2. Find the remainder when $2x^3 - 9x^2 + 14x - 6$ is divided by $x + 2$. (MSHSML 2015-16 1D #2)

$$\begin{array}{r|rrrr} -2 & 2 & -9 & 14 & -6 \\ & \downarrow & -4 & 26 & -80 \\ \hline & 2 & -13 & 40 & -86 \end{array}$$

$$\boxed{-86}$$

$$2x^2 - 13x + 40 \text{ rem } -86$$

$$\begin{array}{r} 2x^2 - 13x \\ x+2 \overline{) 2x^3 - 9x^2 + 14x - 6} \\ \underline{2x^3 + 4x^2} \\ -13x^2 + 14x \\ \vdots \end{array}$$

2. Determine exactly the value of k for which the two solutions of $3x^2 - 4x + k = 0$ are equal. (MSHSML 2014-15 1D #2)

For 2 solns to be equal

$$\begin{aligned} a &= 3 \\ b &= -4 \\ c &= k \end{aligned}$$

$$\text{QF: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac = 0$$

$$(-4)^2 - 4 \cdot 3 \cdot k = 0$$

$$16 - 12k = 0 \quad k = \frac{16}{12} = \boxed{\frac{4}{3}}$$