Math Team

Meet 1 Events C and D Problems #1-2 Practice P2

Event C

Problem #1 ("Quickie"; 1 point)

Try to solve each problem within one minute.

1. In ΔABC, if $\cos A = -\frac{1}{\sqrt{3}}$, determine exactly the value of $\sin A$. (MSHSML 2017-18 1C #1)

$\cos^2\theta + \sin^2\theta = 1$

$\left(-\frac{1}{\sqrt{3}}\right)^2 + \sin^2 A = 1$

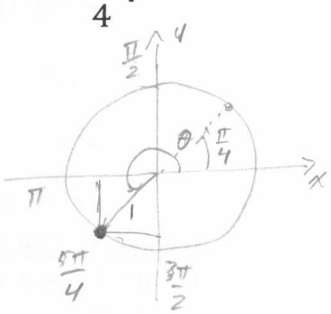
$\frac{1}{3} + \sin^2 A = 1$

$\sin^2 A = \frac{2}{3}$

$\sin A = \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{2\cdot 3}}{3} = \boxed{\frac{\sqrt{6}}{3}}$

1. Determine exactly the value of $\sin\theta + \cos\theta$ if $\theta = \frac{5\pi}{4}$. (MSHSML 2016-17 1C #1)

$\sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\sqrt{2}}$



Problem #2 ("Textbook"; 2 points)

Try to solve each problem within two minutes.

2. For x in radians, $\frac{\pi}{2} < x < \frac{3\pi}{2}$, if $\cot x = 3$, determine exactly the value of $\sec^2 x \cdot \csc x$. (MSHSML 2017-18 1C #2)

$\csc x = \frac{1}{\sin x} = -\sqrt{10}$

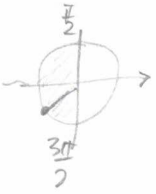
$\frac{10}{9} \quad -\sqrt{10} = \boxed{\frac{-10\sqrt{10}}{9}}$

$\sec x = \frac{1}{\cos x} = \frac{-\sqrt{10}}{3}$

$\cot x = \frac{\cos x}{\sin x} = 3 \Rightarrow \pi < x < \frac{3\pi}{2}$

$\cos^2 x + \sin^2 x = 1$

$\cos x = 3\sin x = \frac{-3}{\sqrt{10}}$



$\cos x = 3\sin x \Rightarrow \cos^2 x = 9\sin^2 x \Rightarrow 1 - \sin^2 x = 9\sin^2 x \Rightarrow 10\sin^2 x = 1 \quad \sin^2 x = \frac{1}{10}$

$\sin x = \frac{-1}{\sqrt{10}}$

2. If $\sin x = \frac{1}{3}$ and $0 < x < \frac{\pi}{2}$, determine exactly the value of $\cos x$. (MSHSML 2016-17 1C #2)

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x + \frac{1}{9} = 1$

$\cos^2 x = \frac{8}{9}$

$\cos x = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{\sqrt{9}} = \frac{\sqrt{4\cdot 2}}{3} = \frac{\sqrt{4}\sqrt{2}}{3} = \boxed{\frac{2\sqrt{2}}{3}}$

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Event D

Problem #1 ("Quickie"; 1 point)

Try to solve each problem within one minute.

1. Determine exactly the remainder when $x^3 - 6x^2 + 4x - 5$ is divided by $x - 3$. (MSHSML 2017-18 1D #1)

$x-3=0 \Rightarrow x=3$ (set)

$(3)^3-6(3)^2+4(3)-5 = 27-54+12-5 = \boxed{-20}$

1. Determine exactly the product of the zeros of the equation $(2x - 7)^2 = 36$. (MSHSML 2016-17 1D #1)

$4x^2-28x+49=36$ 2 roots r_1, r_2

$4x^2-28x+13=0$

$ax^2+bx+c=0$

$r_1 r_2 = \frac{c}{a} = \boxed{\frac{13}{4}}$ Vieta's Formula

$2x-7=\pm 6$

$2x-7=6$ $\quad 2x-7=-6$

$x=\frac{13}{2}$ $\quad x=\frac{1}{2}$

Problem #2 ("Textbook"; 2 points)

Try to solve each problem within two minutes.

Modified from MSHSML problem

× 2. For what values of m does the product of the roots of $4(x - 2m)^2$ equal 11? (MSHSML 2017-18 1D #2) (Modified) (=5 in original)

$4(x^2-4mx+4m^2)=0$

$1x^2-4mx+4m^2=0$

$ax^2+bx+c=0$

$r_1 r_2 = \frac{c}{a} = \frac{4m^2}{1} = 11$ $\quad m^2=\frac{11}{4}$

$\boxed{m=\frac{\pm\sqrt{11}}{2}}$

2. For what value of a does the polynomial $3x^2 + ax + 10$ have 2 as a root? (MSHSML 2016-17 1D #2)

$3x^2+ax+10=0$ For a root, set expression

$3\cdot 4+2a+10=0$

$2a=-22$

$\boxed{a=-11}$