MNJHML Meet 1 Worksheet

Order of Operations, including Invented Operators

1. Determine the value of $3 \cdot 27 - 128 \div 2 + 19 + 2 \cdot 32$

2. If $a \ominus b = 3a - b$, find the value of $7 \ominus (1 \ominus 5)$

3. If $a \Diamond b = a^2 + b^2$, find the value of $(8 \Diamond 1) + (3 \Diamond 4) - (7 \Diamond 4)$

4. Insert parentheses into the following equation to make it true:

$$6 - 4 \cdot 7 + 4 - 3 = 19$$

5. $2 + 3 \times 5 - 7 = 10$, but we can obtain different answers by inserting parentheses, for instance $2 + 3 \times (5 - 7) = -4$.

   a. Including 10 and -4, how many different values can be obtained by inserting a single set of parentheses into the expression $2 + 3 \times 5 - 7$?

   b. Implied multiplication is not permitted, so for instance $2 + 3 \times 5(-7)$ is not considered a valid expressions for the purposes of this problem.

Rational Number Concepts including Decimals and Fractions

6. What is the value of $\frac{3}{10} + \frac{2}{5} - \frac{1}{15}$? Express your answer as a common fraction.

7. 1.2.2. Evaluate $\frac{4}{0.2} + \frac{0.6}{0.12}$.

8. A log cabin can be built using the lumber from 2 ⅔ trees. If an acre of forest contains 150 trees, how many cabins can be built using the wood contained in a four-acre forest?

9. Express the following sum of rational numbers as a decimal:

$$2 + \frac{3}{5} + \frac{7}{20}$$

10. 1.2.5. Simplify the expression. Express your answer as a ratio of relatively prime integers:

$$1 + \frac{2}{2+\frac{2}{3+\frac{2}{4}}}$$

Positive Exponents and Roots

11. Evaluate 2^6 - 6^2.

12. When multiplied out, what is the sum of the digits in the number $7^2 \cdot 2^{2017} \cdot 5^{2018}$?

13. Simplify: $\sqrt{3^5 \cdot 16 \cdot 6}$

14. 1.3.3. What is the value of n for which $\sqrt[n]{27^7 \cdot 3^{15} \cdot 81^4} = 9$?

15. 1.3.5. If $(6^3)^{11} = x \cdot (x^4)^8$, what value for x makes the statement true?

Speed and Other Rates (Distance = Rate × Time)

16. Deep Blue, the first computer chess program to defeat a human World Champion, evaluates 200 million positions per second. How many billion positions does it evaluate during a one-hour match?

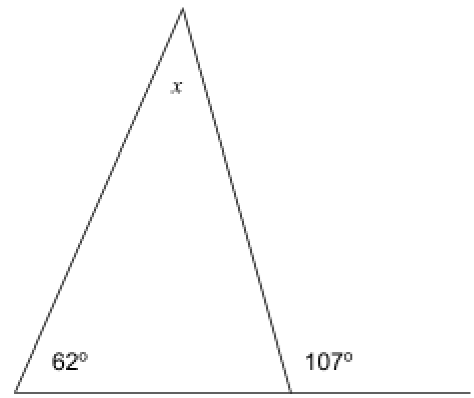
17. The tortoise and the hare run a 400m race. The tortoise moves at a steady rate of ½ meter per second while the hare runs at 5 meters per second. 300m into the race the hare is far ahead and stops to take a short nap. After a while it wakes up to see the tortoise nearing the finish. The hare immediately starts running and both animals arrive at the finish line at the same time. How many minutes did the hare nap?

18. A rower moves at a constant rate of 2 miles per hour in still water. She launches her boat into a river flowing at 1 mile per hour, then rows 40 minutes into the current, turns around, rows 20 minutes with the current, and pulls her boat to shore. How far must she walk, in miles, to return to the point from which she launched? Express your answer as a common fraction.

19. 1.4.4. Hafsa and Hoda live 4 miles away from each other on the same street. Hafsa leaves her house and runs toward Hoda's house. At the same time, Hoda leaves her house and runs toward Hafsa's house. If Hafsa runs at three-fifths the rate that Hoda runs, how far from Hafsa's house will they meet each other? Express your answer as a decimal rounded to the nearest tenth of a mile.

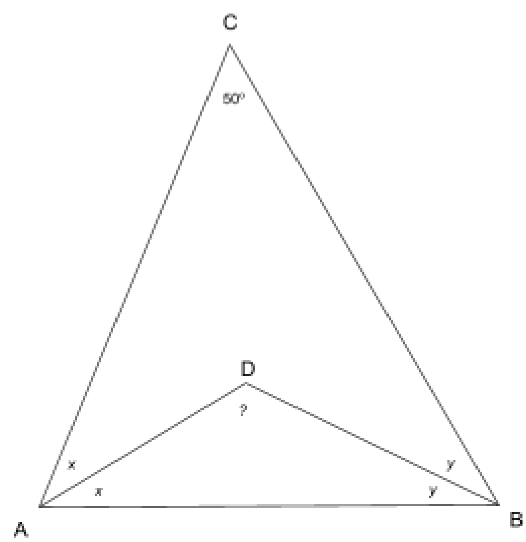
Finding Angle Measures

20. What is the value of x in the figure shown?

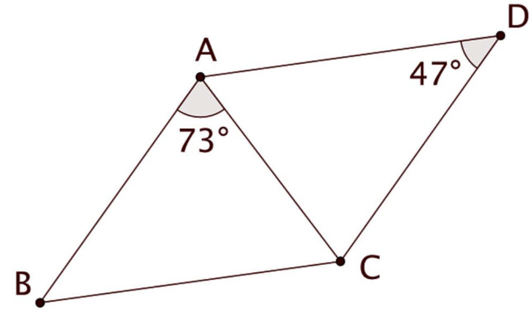


21. What is the measure of the acute angle formed by the hands of a clock at 6:40 PM?

22. In the diagram shown, segment *AD* bisects angle *CAB* and segment *BD* bisects angle *ABC*. What is the measure of angle *ADB*?



23. If $ABCD$ is a parallelogram, find the measure of angle BCA.



Multiplication Principle and Counting with Permutations

24. How many four-digit positive integers are there with a hundreds digit of 2?

25. At the school cafeteria, students choose between milk, juice, or water to drink, a turkey sandwich, veggie wrap, or pizza slice as an entree, and carrot sticks or tossed salad as a vegetable. Every student must select exactly one drink, one entree, and one vegetable. How many different lunch combinations are possible?

26. Maya creates a four-digit security code with different digits using only the numbers 1, 2, 3, 4, 5, and 6. How many such codes are possible?

27. Car owners in Minnesota can pay an extra $25 per year in licensing fees to get special collegiate license plates for their vehicles. The U of M plate pictured uses two letters and three numbers. How many unique license plates of this type are possible, if the letters I, O and Q can **not** be used?

28. At this meet, Events A and B each contain 5 problems. The first three problems in each event are worth two points each, while the last two problems are worth four points each. How many different **Event B** papers could be made from the same set of 10 problems (6 two-point and 4 four-point problems)? The point values attached to each problem must remain the same, and an event with the same five problems in a different order would be considered different.

Challenge Problem

29. A magic square is an array of the numbers 1, 2, 3, … such that the sum of the numbers in any row, column, or main diagonal line is always the same number.

    What number should be placed in the shaded cell of the partially completed 4x4 magic square below?

1	15		
		8	5
	6	9	12
16			