SMacLennan
Tu 11/2/2020
ERA Sar MT
Form AB

Math Team
Meet 1 Events A and B Problems 1-2 2018-20 Practice

Event A

Problem #1 ("quickie"; 1 point)

Try to solve each problem within one minute.

1. Find the sum of the two solutions to this absolute value equation: $|2x + 1| = 9$. [calculator allowed] (MSHSML 2019-20 2A #1)

$2x+1=-9$ or $2x+1=9$

$2x=-10$ $\quad 2x=8$

$x=-5$ $\quad x=4$

$x=-5,4$ $\quad$ sum: $-5+4=\boxed{-1}$

1. Sal earns \$30.00 for a day's work but also receives a commission of 5% on all the merchandise she sells. If she earned \$120.00 yesterday, how much merchandise did she sell? [calculator allowed] (MSHSML 2018-19 2A #1)

Let $x=$ \$

$120-30=\$90$ commission

$(0.05)x=90$

$x=\boxed{1800}=\boxed{\$1800}$

Problem #2 ("textbook"; 2 points)

Try to solve each problem within two minutes.

2. I have nickels, dimes, and quarters in my pocket. The total of this change is \$3.90. I have twice as many nickels as dimes and half as many quarters as one of the other coins. How many dimes do I have? [calculator allowed] (MSHSML 2019-20 2A #2)

$n = 2d$

$5n + 10d + 25q = 390$

$10d + 10d + 25q = 390$

$20d + 25q = 390$

Case 1: $20d + 25\left(\frac{n}{2}\right)$

$20d + 12.5n$

$20d + 25d = 390$

$45d = 390$ ×

d not an integer

Case 1: $q \stackrel{?}{=} \frac{n}{2}$ ×

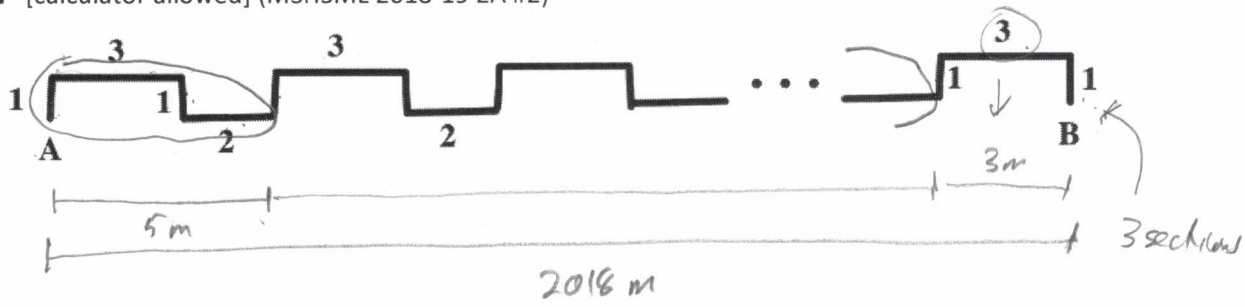
Case 2: $q = \frac{d}{2}$

Case 2: $20d + \frac{25d}{2} = 390$

$32.5d = 390$

$d = 12$ dimes

2. The figure below is a portion of a highway wall as seen from above. The vertical sections are each 1 meter wide, the upper horizontal sections are each 3 meters wide, and the lower horizontal sections are 2 meters wide. If the length of the wall, i.e., the straight-line distance from A to B is 2018 meters, how many total sections are there in the wall? [calculator allowed] (MSHSML 2018-19 2A #2)



$\frac{2015}{5} = 403$

403 (4 sections) $= 1612$

Total: $1612 + 3 = \boxed{1615}$

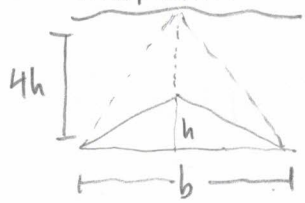
Event B

Problem #1 ("quickie"; 1 point)

Try to solve each problem within one minute.

1. When the height of a triangle is quadrupled (made four times larger), its area increased by 2019. What is the area of the original triangle? [calculator allowed] (MSHSML 2019-20 2B #1)

Interpretation 1

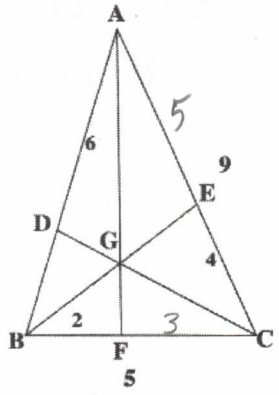


$\frac{1}{2}b(4h) - \frac{1}{2}bh = 2019$

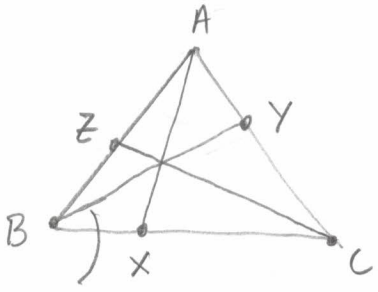
$2bh - \frac{1}{2}bh = 2019$

$\frac{\frac{3}{2}bh}{3} = \frac{2019}{3} \Rightarrow \frac{1}{2}bh = \boxed{673}$

1. In $\triangle ABC$ at the right, $AC = 9$ and $BC = 5$. Segments $\overline{BE}$, $\overline{CD}$, and $\overline{AF}$ are concurrent at G. If $BF = 2$, $CE = 4$, and $AD = 6$, determine exactly DB. [calculator allowed] (MSHSML 2018-19 2B #1)



"Ceva's theorem!"



Cevian

$\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$

$\Downarrow$

$\frac{AE}{EC} \cdot \frac{CF}{FB} \cdot \frac{BD}{DA} = 1$

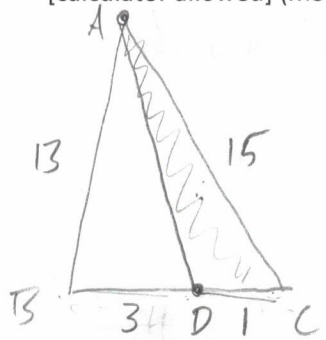
$\frac{5}{4} \cdot \frac{3}{2} \cdot \frac{BD}{6-BD} = 1$

$BD = \frac{4 \cdot 2 \cdot 2}{5} = \boxed{\frac{16}{5}}$

Problem #2 ("textbook"; 2 points)

Try to solve each problem within two minutes.

2. In $\triangle ABC$, $AB = 13$, $BC = 4$, and $CA = 15$. Cevian $\overline{AD}$ is drawn such that $CD = 1$. Determine exactly $[ADC]$.[1] [calculator allowed] (MSHSML 2019-20 2B #2)



Footnote 1

$[ADC]$ / area of polygon (triangle) ADC

Heron's Formula for $\triangle ABC$

$s = \frac{1}{2}(13+15+4) = \frac{1}{2}(32) = 16$

$[ABC] = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(3)(1)(12)}$

$= \sqrt{16 \cdot 36} = 4 \cdot 6 = 24$

$[ADC] = \frac{1}{3+1}[ABC]$

$= \frac{1}{4}(24)$

$= \boxed{6}$

2. What is the area of a triangle with side lengths 25, 25, and 48? [calculator allowed] (MSHSML 2018-19 2B #2)

[1] The notation $[ABC]$ indicates the area of the polygon (a triangle, in this example) ABC.