S MacLennan
Th 5 Nov 2020
ERA 80R Math Team
Forum CD

Math Team
Meet 2 Events C and D Problems #1-2 Practice 2018-20

Event C

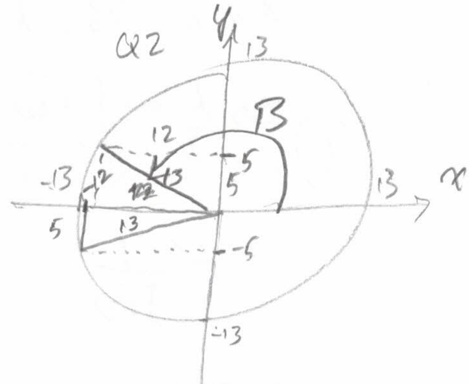
Problem #1 ("Quickie"; 1 point)

Try to solve each problem within one minute.

1. If $\frac{\pi}{2} < B < \pi$ and $\sin B = \frac{5}{13}$, determine exactly $\sin(2B)$.

(MSHSML 2019-20 2C #1)

$\cos B = -\frac{12}{13}$



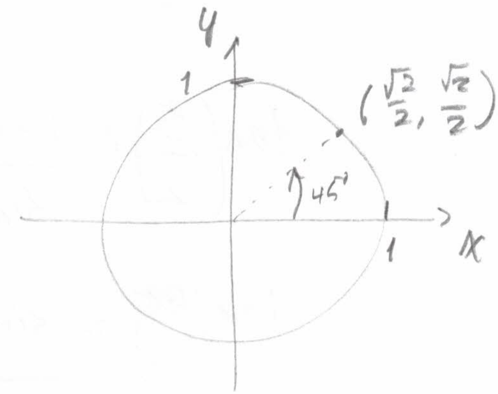
$\sin 2B = 2 \sin B \cos B$

$= 2 \cdot \frac{5}{13} \cdot \left(\frac{-12}{13}\right) = \boxed{\frac{-120}{169}}$

1. Determine exactly the smallest possible positive degree measure for θ, given that $\tan 9\theta = 1$. (MSHSML 2018-19 2C #1)

$\tan\theta = \frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \sin x = \cos x$

$\Rightarrow x = 45°$



$x = 9\theta$

$9\theta = 45°$

$\theta = \boxed{5°}$

Problem #2 ("Textbook"; 2 points)

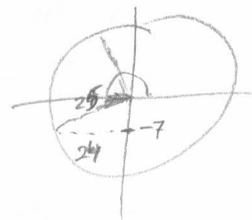
Try to solve each problem within two minutes.

2. If $\pi < A < \frac{3\pi}{2}$ and $\sin A = -\frac{7}{25}$, determine exactly $\cos\frac{A}{2}$.

(MSHSML 2019-20 2C #2)

A is in Q3, $A \in Q3$

$\cos A = -\frac{24}{25}$

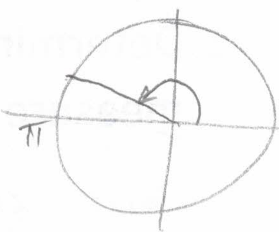


$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} = -\sqrt{\frac{1+\left(-\frac{24}{25}\right)}{2}} = -\sqrt{\frac{25-24}{50}} = -\sqrt{\frac{1}{50}}$$

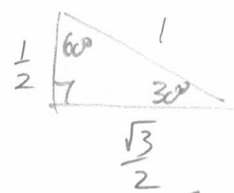
$$= -\frac{1}{\sqrt{50}} = -\frac{1}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{10}}$$

2. Determine exactly the value of $\tan\frac{5\pi}{12}$. (MSHSML 2018-19 2C #2)

$$\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{\frac{5\pi}{6}}{2}\right)$$



$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$



$$\tan\left(\frac{5\pi}{12}\right) = \frac{\sin\frac{5\pi}{6}}{1+\cos\frac{5\pi}{6}} = \frac{\frac{1}{2}}{1+\left(-\frac{\sqrt{3}}{2}\right)} = \frac{1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3}$$

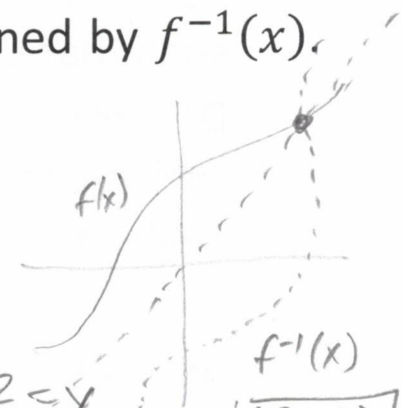
$$= \boxed{2+\sqrt{3}}$$

Event D

Problem #1 ("Quickie"; 1 point)

Try to solve each problem within one minute.

1. Determine exactly the point of intersection of the line defined by $f(x) = \frac{3x+2}{6} = y$ and the line defined by $f^{-1}(x)$. [calculator allowed] (MSHSML 2019-20 2D #1)



Long way: $f^{-1}(x)$: $\frac{3y+2}{6} = x$

Short way: know that $f(x)$, $f^{-1}(x)$ intersect on the line $y = x$

$y = \frac{3x+2}{6} \Rightarrow x = \frac{3x+2}{6} \Rightarrow 6x = 3x+2 \Rightarrow x = \frac{2}{3} = y$

$\therefore \boxed{\left(\frac{2}{3}, \frac{2}{3}\right)}$

1. Calculate the slope of the line $8x + 11y - 13 = 0$. [calculator allowed] (MSHSML 2018-19 2D #1)

$8x + 11y - 13 = 0$

$\frac{11y}{11} = \frac{-8x + 13}{11}$

$y = -\frac{8}{11}x$

$m = \boxed{\frac{-8}{11}}$

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Problem #2 ("Textbook"; 2 points)

Try to solve each problem within two minutes.

2. Let l_1 be the line $5x - 4y = 9$ and l_2 be the line $10x -$ $- Ay = 2$, where A is a constant. There is one value for A such that $l_1 \parallel l_2$ and another value for A such that $l_1 \perp l_2$. Determine exactly the product of these two values of A. [calculator allowed] (MSHSML 2019-20 2D #2)

Standard form: $Ax + By = C$ or $Ax + By + C = 0$

our problem $\Rightarrow$ slope $= \frac{-A}{B}$

$A_1 \cdot A_2 = 8(-12.5)$

$= \boxed{-100}$

$M_1 = \frac{-5}{-4} = \frac{5}{4}$ $\quad m_2 = \frac{-10}{-A} = \frac{10}{A}$

Parallel: $m_1 = m_2$ $\quad \frac{5}{4} = \frac{10}{A} \Rightarrow A_1 = 8$ $\quad$ Perp: $m_1 = -\frac{1}{m_2} \Rightarrow \frac{5}{4} = \frac{-A}{10} \Rightarrow 50 = -4A$

$A_2 = -12.5$

2. Determine exactly, in the form $Ax + By = C$, the equation of the line with a negative slope that contains the center and the y-intercept of the circle $(x-4)^2 + (y-5)^2 = 65$. [calculator allowed] (MSHSML 2018-19 2D #2)

[Did not do. You can do it!]