MNJHML Meet 5 Answers

1. 99
2. b=6
3. c=5
4. 402
5. 38 cards
6. x+y=20
7. (3,9)
8. $x = \frac{9}{4}$
9. 16
10. r=16
11. 3
12. 4
13. $|XY| = \frac{15}{4}$
14. 20,000 liters of water
15. $y = \frac{3}{4}x$
16. $\frac{9}{19}$
17. $\frac{3}{8}$
18. $\frac{4}{9}$
19. $\frac{3}{8}$
20. 43%
21. $\frac{1}{3}$
22. 50 degrees

MNJHML Meet 5 Complete Solutions

1. Converting both numbers to base 10, we have:

$$1001_2 \times 13_8$$
$$= (1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1) \times (1 \times 8^1 + 3)$$
$$= 9 \times 11$$
$$= 99$$

2. Convert to base ten:

$$62_9 = 132_b$$
$$6 \cdot 9 + 2 = b^2 + 3b + 2$$
$$54 = b^2 + 3b$$
$$54 = b(b + 3)$$

Since b is a positive integer less than 9, we can guess and check to see that $b = 6$.

3. Proceed with long division in base 9:

```
  0.1251...
7 ) 1.0000000
    7
    20
    15
     40
     38
      10
      ...
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The key is to do all operations in base 9, e.g. $10_9 - 7_9 = 2_9$, $7_9 \times 2_9 = 15_9$, etc.

Then we see that in this base 9 representation, $c = 5$.

4. Rewrite as an addition problem:

$234_n + 135_n = XX2_n$.

From the units digit, since $4_n + 5_n = 2_n$ isn't true for any base n, we must have $4_n + 5_n = 12_n$. Converting to base 10 gives the equation $4 + 5 = n + 2$, so $n = 7$. Now that we know the base we can proceed as a base-7 addition problem:

```
  1
 X34
+135
 X02
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(carry the 1, add, and the second digit is 0 since $1_7 + 3_7 + 3_7 = 10_7$)

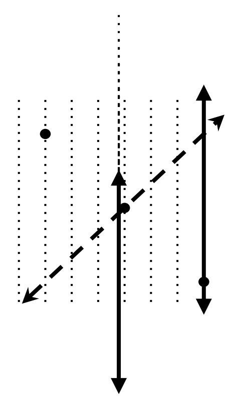
```
 1 1
 234
```

+135
 402
 (carry the 1, add, and the first digit is 4, so the answer is 402).

5. Suppose Anya has m Pokemon trading cards and Adeem has n cards.
   Then the conditions of the question tell us that:
   $$m + n = 96$$
   and $$m = 1.5n$$
   Substituting $m = 1.5n$ into $m + n = 95$:
   $$1.5n + n = 95$$
   $$2.5n = 95$$
   $$5n = 190$$
   $$n = 38$$

Therefore Adeem has 38 Pokemon cards.

6. We could proceed by substitution or elimination, but there's a nice symmetry between x and y in the equations which suggests adding both equations:
   $$(x\ +\ 4y)\ +\ (4x + y)\ =\ 100$$
   $$5x\ +\ 5x\ =\ 100$$
   so $$x\ +\ y\ =\ 20$$
   This answers the questions without even needing to find x or y!
7. First, which points are equidistant from
   Clearly the midpoint,
   $(-3,3)$ is equidistant. In fact with some thought we see that
   any point along the dashed line shown is equidistant
   (whereas other points are closer to either one or the other).
   This line has a y-intercept of 6 and a slope of 1, so it's
   equation is $y = x + 6$.
   But then the point *P* must satisfy both equations:



   $$y\ = 15 - 2x$$
   $$y = x + 6$$
   Therefore $15 - 2x = x + 6$ so $x = 3$ at the point of intersection, and substituting back $y = 9$. So *P* is the point $(3,9)$ which is equidistant from $(0,0)$ and $(-6,6)$ and is also on the line $y = 15 - 2x$.
   $(0,0)$ and $(-6,6)$? A diagram may help:
8. We carefully distribute the left hand side:
   $$3x(x - 9) + 7(x - 9) = 3x^2 - 108$$
   $$3x^2 - 27x + 7x - 63 = 3x^2 - 108$$
   $$-20x - 63 = -108$$
   $$-20x = -45$$
   $$x = \frac{45}{20}$$

So we conclude that $x = \frac{9}{4}$.

9. Resist the urge to multiply out the expressions in the numerator and denominator: it will take a long time and almost certainly result in an error somewhere!
   Instead recall that $(m+m)^2 = m^2 + 2mn + n^2$ for any m and n, so the numerator can be written as just $(732 + 268)^2 = 1000^2$.
   Similarly $(m-m)^2 = m^2 - 2mn + n^2$ for any m and n, so the denominator equals simply $(387 - 137)^2 = 250^2$.
   Putting these together, our original fraction evaluates to:
   $$\frac{1000^2}{250^2} = \left(\frac{1000}{250}\right)^2 = 4^2 = 16$$
   So this expression has a value of 16.
10. Distribute the left hand side (or recognize the difference of squares identity):
   $$r(r+9) - 9(r+9) = 8r + 47$$
   $$r^2 + 9r - 9r - 81 = 8r + 47$$
   $$r^2 - 81 = 8r + 47$$
   $$r^2 - 8r - 81 - 47 = 0$$
   $$r^2 - 8r - 128 = 0$$
   This factors as
   $$(r-16)(r-8) = 0$$
   So $r = 16$ or $r = 8$ are the two solutions. Then the largest value of r satisfying the original equation is 16.
11. This factors as $(x-3)(x-3) = 0$ (or $(x-3)^2 = 0$), so $x = 3$ is the only solution, and the sum of the solutions to this equation is 3.
12. Not having a better idea let's assign a variable:
   $$x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}$$
   Then
   $$x = \sqrt{12 + \left(\sqrt{12 + \sqrt{12 + \ldots}}\right)}$$
   Note the expression in parentheses is also just x, so we have the equation
   $$x = \sqrt{12 + x}$$
   Squaring both sides gives
   $$x^2 = 12 + x$$

Or, in standard form

$$x^2 - x - 12 = 0$$

Factoring this gives

$$(x-4)(x+3) = 0$$

So either $(x-4) = 0$ or $(x+3) = 0$. The latter doesn't make sense since clearly $x > 0$, so we must have $(x-4) = 0$, in other words $x = 4$ so the value of this infinite expression must be 4.

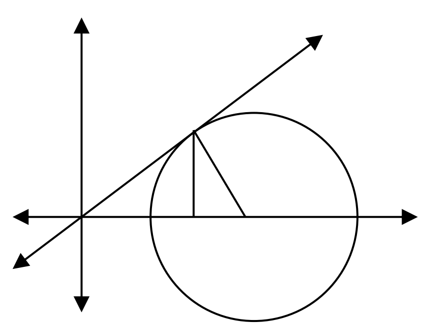
13. Notice that triangles XYC and ABC are similar since all three corresponding angles are equal, so therefore their sides share a common ratio.
    In particular $|XY|:|YC| = |AB|:|BC|$
    Substituting known side lengths gives

    $$|XY|:3 = 10:8$$

    or $\frac{|XY|}{3} = \frac{10}{8}$

    $|XY| = \frac{3\times 10}{8}$

    $|XY| = \frac{15}{4}$
14. Since her backyard pool was built using the same proportions as the original and its length is ⅕ as long, it must also be ⅕ as wide and ⅕ as deep. This means that it's volume is ⅕ × ⅕ × ⅕ or $\frac{1}{125}$ as much as the original.
    Since an Olympic sized pool contains 2,500,000 liters of water, Putri's pool requires $\frac{1}{125} \times 2{,}500{,}000 = 20{,}000$ liters of water.
15. We label some points and draw some additional lines: label O at the origin, A(0,5) as the circle's center, and B as the point where the line is tangent to the circle. Draw in segment AB and draw a perpendicular from B to OA at C.
    Now, |OA|=5 and |AB|=3, so since OAB is a right triangle by the Pythagorean Theorem (or recognizing a special right triangle) we can see that OB=4.
    To find the equation of our line we'd like to know its slope, which is $\frac{|BC|}{|OC|}$.



But notice that triangles OCB and OBA are similar (both share a common angle and have a right angle), so we must have:

$$\frac{|BC|}{|OC|} = \frac{|AB|}{|OB|} = \frac{3}{4}$$

Then we see that our line has a slope of $\frac{3}{4}$ and a y-intercept of 0, so its equation is

$$y = \frac{3}{4}x$$

16. 10 of the 20 socks in the drawer are black, so the probability that the first sock drawn is black will be $\frac{1}{2}$. Once this sock has been drawn, 9 of the 19 remaining socks are black, so the probability that the second sock drawn is black is $\frac{9}{19}$.
    By the multiplication principle, the probability that both socks drawn are black is $\frac{1}{2} \times \frac{9}{19} = \frac{9}{38}$.
17. There are two possible ways to get a positive product: either both numbers spun are positive, or both numbers spun are negative. Since they're exclusive events we can add the two probabilities.
    The probability that both numbers spun are positive is $\frac{2}{4} \cdot \frac{2}{4} = \frac{1}{4}$ (since there are two positive numbers on each spinner).
    The probability that both numbers spun are negative is $\frac{1}{4} \cdot \frac{2}{4} = \frac{1}{8}$ (one negative number on the first spinner, two on the second).
    Overall, this gives a $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ probability that the product of the numbers spun is positive.
18. Consider the center of the quarter. If it lands more than 0.5" from any line, then the quarter isn't touching any of the lines (and otherwise it is). In other words, if the center of the quarter is within the central 2"×2" square inside each 3"×3" grid square then it won't be touching a line.
    Then since the quarter lands randomly within some square, the probability that it doesn't touch any line is the same as the ratio of the area of the inner 2"×2" square to the area of the full 3"×3" square, which is $\frac{4}{9}$.
19. In total, there are 4! = 24 ways to randomly assign the four bags to the four players (four choices for the first player, then 3 for the second, 2 for the third, and the fourth player gets the remaining bag).
    For notational purposes, we'll designate an assignment by a four digit number where the first digit indicates which bag the first player got, the second shows which bag the second player got, and so on. For instance 1432 would indicate that players 1 and 3 got their own bags, but players 2 and 4 got each-other's bag.
    Using this notation we can carefully list assignments in which no player gets their own bag. We list these in numerical order to make sure we don't miss any:

2143   2341   2413   3142   3412   3421   4123   4312   4321

This is 9 arrangements in which no player got their own bag, so the probability that no player gets the correct bag is $\frac{9}{24}$ or $\frac{3}{8}$.

20. Since 44% of electricity was generated from Coal and 13% from Natural Gas, in total 57% of electricity was generated from fossil fuels.

    Then the remaining 100%-57% or 43% of electricity was generated from sources other than fossil fuels.

21. Reading the graph we see that 15 students scored 0 goals, 6 scores 1 goal, 13 scored 2 goals, 5 scored 3 goals, 2 scored 4 goals, and 4 scored 5 goals.

    In order by goals scored, the 23rd student scored 2 goals, so the median number of goals scored per student is 2.

    To find the mean, we add up the number of goals scored by all students. We'll have 15 students adding 0 goals each, 6 adding 1 goal each, and so on, so the total number of goals scored by all 45 students is

$$15 \times 0 + 6 \times 1 + 13 \times 2 + 5 \times 3 + 2 \times 4 + 4 \times 5$$
$$= 0 + 6 + 26 + 15 + 8 + 20$$
$$= 75$$

    Then the average number of goals scored was $\frac{75}{45} = \frac{5}{3}$.

    Now we can see that the median number of goals scored (2) is $\frac{1}{3}$more than the mean number of goals scores ($\frac{5}{3}$), so the answer is $\frac{1}{3}$.

22. There are 360º in a circle, so with a total population of 10,800 every degree in the circle will represent $\frac{10800}{360} = 30$ people.

    Then with 1,500 people in Lake County in the 10-19 age category, this segment of the pie chart should have a central angle of $\frac{1500}{30} = 50$ degrees.