S Maclennan
SoL Math Team
W 11 Nov 2020

MNJHL Meet 2 Worksheet

W 11/11

1. What is the smallest prime number which becomes a composite number when its digits are reversed?

prime: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 → 31, 71, 91 = 7×13 → **19**

(composite: a positive integer that is not prime)

2. How many positive integers divide 140?

$140 = 2^2 \cdot 5 \cdot 7$, $3 \cdot 2 \cdot 2 = 12$ → **12**

3. Evaluate $\sqrt{216 \times 294}$

$= \sqrt{2^3 \cdot 3^3 \cdot 2 \cdot 3 \cdot 7^2} = \sqrt{2^4 \cdot 3^4 \cdot 7^2} = 2^2 \cdot 3^2 \cdot 7 =$ **252**

4. The five digit number $n = 4\_72\_$ is divisible by *18* (where _ represent unknown digits of n). What is the smallest possible value of n?

$18 = 2 \cdot 3 \cdot 3$; 4A72B, B even ⇒ B ∈ {0,2,4,6,8}; 4+A+7+2+B is a multiple of 9, A+B = 5 or 14 → **41,724**

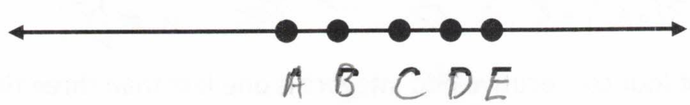
5. A palindrome is a number which has the same value when its digits are reversed. For example, 3113 is a palindrome. If P and Q are two-digit palindromes, what is the smallest possible product PQ that is not a palindrome?

P = 11p, Q = 11q, PQ = 11p · 11q = 121pq = 121, 242, 363, 484, **605**

6. What number is midway between $-1\frac{1}{2}$ and $\frac{3}{4}$ on the number line?

$\frac{3}{4} - (-1\frac{1}{2}) = \frac{9}{4}$ distance between; $\frac{9}{8}$ from either point: $\frac{3}{4} - \frac{9}{8} = \frac{6}{8} - \frac{9}{8} =$ **$-\frac{3}{8}$**

7. The numbers $\frac{5}{3}, \sqrt{3}, \frac{7}{4}, \frac{3}{2}$, and $\sqrt{2}$ are plotted on the real number line in their correct order below. Which number is represented by point D?



$2 < \frac{9}{4} < \frac{25}{9} < 3 < \frac{49}{16}$ ∴ D = **$\sqrt{3}$**

8. An ant and a spider play a game on the real number

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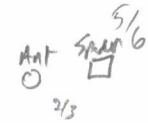
8. An ant and a spider play a game on the real number line. They both start at the origin (0) and take turns moving: first the ant moves to the number of its choice, then the spider moves halfway towards the ant.

If the ant moves to 3, -2, then ⅔ for its first three moves, at what number does the spider end up after its third move? Express your answer as a common fraction.

5/6

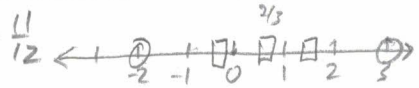


Ant → 3 → −2 → 2/3

Spider → 1½ → −¼ → 5/24

$-\frac{6}{24} + \frac{11}{24}$

$\frac{2}{3} - (-\frac{1}{4}) = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$



$= \frac{22}{24}$ ½ is $\frac{11}{24}$

9. What is the positive difference between the absolute value of −13 and the opposite of −7?

$\big||-13| - 7\big| = |13-7| = |6| = \boxed{6}$

6

10. Solve for a:

check: $-8 + (+16) + (+16) \stackrel{?}{=} 24$ ✓

$$a - (a + (a - a) + a) - (a + a) = 24$$

$a - (a + 0 + a) - (a+a)$

$1a - 2a - 2a = 24$

$-3a = 24$

$\frac{-3a}{-3} = \frac{24}{-3}$

$a = \boxed{-8}$

NS

11. Solve for g:

$$3g - 4 = 6 - 2g + 2(1 - g)$$

Express your answer as a common fraction.

$3g - 4 = 6 - 2g + 2 + (-2g)$

$3g - 4 = 8 - 4g$

$+4g \quad +4 \quad +4 \quad +4g$

$7g = 12$

$\frac{7g}{7} = \frac{12}{7}$

preferred

$g = \boxed{\frac{12}{7}} = \boxed{1\frac{5}{7}}$

1 5/7

12. Mateo takes a number, adds 1 to it, multiplies the answer by 3, subtracts 5 from the result, and finally divides that number by 2. If his answer is 11, what was the original number?

$\frac{3(n+1)-5}{2} = 11 \cdot 2$

$3(n+1) - 5 = 22$

$3n + 3 - 5 = 22$

$3n - 2 = 22$

$3n = 24$

$n = \boxed{8}$

8

13. The sum of four consecutive odd integers is one less than three times the smallest number. What is the sum of the four integers?

$m + (m+2) + (m+4) + (m+6) = 3m - 1$

$4m + 12 = 3m - 1$

$-3m \quad -12 \quad -3m \quad -12$

$m = -13$

$-13 + (-11) + (-9) + (-7) = \boxed{-40}$

-20

-20

-13
-11
-9
-7

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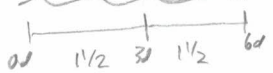
14. Two brothers can eat three watermelons in six days. How long would it take one of them to eat two watermelons (assuming both brothers eat at the same rate)?



1½ in 6 days each ⇒ ¼ per day

2 watermelons in $\frac{2}{1/4}$ = 8 days

15. The Jolly Green Giant statue in Blue Earth, MN is 56 feet tall. A tourist shop nearby sells 7 inch tall scale models of the original. If the actual statue's feet are 6 feet in length, how long are the feet on the toy? Express your answer in inches as a common fraction.

Let f = foot of toy model in inches

$\frac{56}{6} = \frac{7}{f}$ $\quad 56f = 42$

$f = \frac{42}{56} = \frac{21}{28} = \boxed{\frac{3}{4}}$

W 11/18

16. Two 500ml bottles contain olive oil; one is ¾ full and the other is ⅓ full. In order to make a vinaigrette salad dressing, balsamic vinegar is added to fill each bottle then the two are poured into a single larger 1 liter bottle. What fraction of this larger bottle contains vinegar? Express your answer as a common fraction.

add vinaigrette $\frac{1}{4}$ $\quad \frac{1}{3}$

$\frac{\frac{1}{4}+\frac{1}{3}}{2} = \frac{1}{2}\left(\frac{1}{4}+\frac{1}{3}\right) = \frac{1}{2}\left(\frac{3}{12}+\frac{4}{12}\right)$

$= \frac{1}{2}\left(\frac{7}{12}\right) = \boxed{\frac{7}{24}}$

17. The measures of the angles in a triangle are in the ratio 1:3:5. What is the measure of its largest angle?

↳ 1+3+5 = 9 ⇒ $\frac{180}{9}$ = 20° per part

sum of angles in a triangle theorem: sum is 180° (9 parts)

ratio is 20:60:100 ⇒ largest angle is $\boxed{100°}$

18. What is the circumference of a circle whose area is 121π square inches? Express your answer in terms of π.

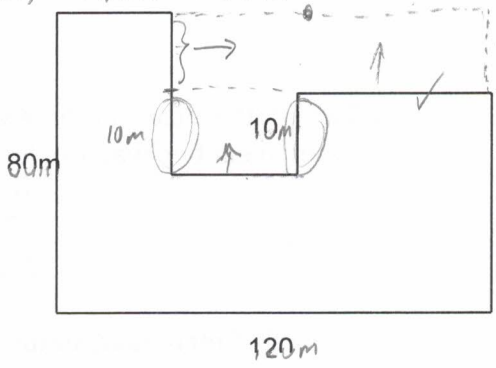
area = $\pi r^2 = 121\pi \Rightarrow r^2 = 121 \Rightarrow r = 11$ in

$C = 2\pi r = 2\pi(11) = \boxed{22\pi \text{ inches}} = \boxed{22\pi \text{ in.}}$

19. Three sides of the school building shown are 80 meters, 10 meters, and 120 meters in / cutoff!

[see next page]

19. Three sides of the school building shown are 80 m, 10 m, and 120 m in length. If all walls meet at right angles, what is the perimeter of this building?

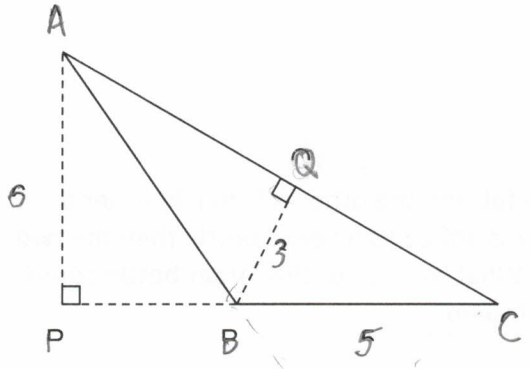


$P = 2(80+120) + 2(10)$

$= 2(200) + 20 = 400+20$

$= \boxed{420\text{ m}}$

20. In the figure shown, determine the length of side *AC*.



area of $\triangle ABC = \frac{1}{2}bh = \frac{1}{2}(5)(6) = 15$

$= \frac{1}{2} AC \cdot 3 = \frac{3}{2} AC$

$\therefore \frac{3}{2} AC = 15$

$\boxed{AC = 10}$

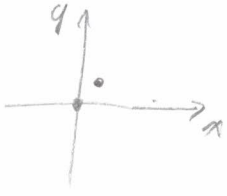
21. A mathematically inclined bug plays the following game on the coordinate plane: when it finds itself at the point (x, y) it moves to the point $(x + 1, 2x + y + 1)$. For instance from the point $(3, 2)$ it would move to $(4, 9)$.
If the bug starts out at $(0, 0)$, where will it be after 12 moves? Express your answer as an ordered pair.

x y

Move 1 → Move 6

$(0,0) \to (1,1) \to (2,4) \to (3,9) \to (4,16) \to (5,25) \to (6,36)$

$\to (7,49) \to (8,64) \to (9,81) \to (10,100) \to (11,121) \to \boxed{(12,144)}$



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22. A Lattice Point in the coordinate plane is a point (m, n) where both m and n are integers. For instance (4, -7) is a Lattice Point but (3, 0.5) is not.

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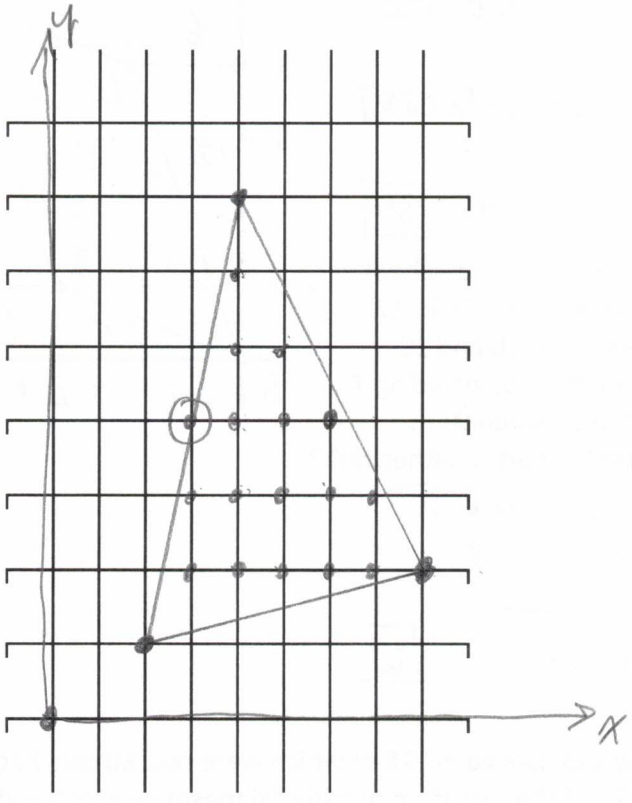
A triangle is drawn with vertices at (2, 1), (8, 2), and (4, 7). Including these three points, how many Lattice Points are contained within or on the perimeter of this triangle?



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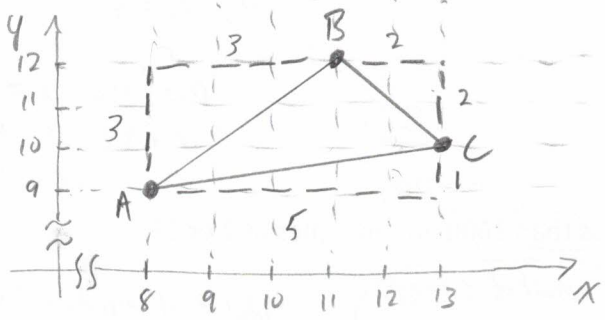
23. Determine the area of triangle ABC with vertices $A(8,9)$, $B(11,12)$ and $C(13,10)$



$AB = 3\sqrt{2}$, $BC = 2\sqrt{2}$ $AC = \sqrt{26}$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 8 & 11 & 13 \\ 9 & 12 & 10 \\ 1 & 1 & 1 \end{vmatrix}$$

$\frac{1}{2}$ base · height

$$[\triangle ABC] = 3\cdot 5 - \tfrac{1}{2}3\cdot 3 - \tfrac{1}{2}\cdot 2\cdot 2 - \tfrac{1}{2}\cdot 5\cdot 1$$

$$= 15 - \frac{9}{2} - 2 - \frac{5}{2}$$

$$= 15 - 2 - \frac{14}{2} = 15 - 2 - 7$$

$$= 15 - 9 = \boxed{6}$$

$\sqrt{18} = \sqrt{9\cdot 2} = \sqrt{9}\,\sqrt{2} = 3\sqrt{2}$

$\sqrt{8} = \sqrt{4\cdot 2} = \sqrt{4}\cdot\sqrt{2} = 2\sqrt{2}$

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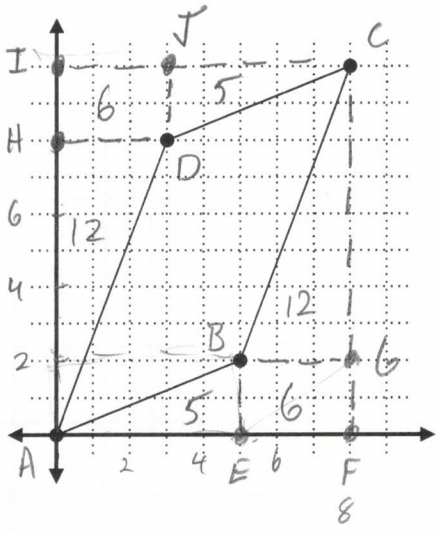
24. Determine the area of the quadrilateral ABCD shown where A is (0,0), B is (5,2), C is (8,10), and D is (3,8).



Area of ABCD = [ABCD] =

= [AICJ] − [△AEB] − [EFGB] − [△BGC] − [△CJD] − [JIHD] − [△HAD]

= 10·8 − 5 − 6 − 12 − 5 − 6 − 12 = 80 − 46 = **34**

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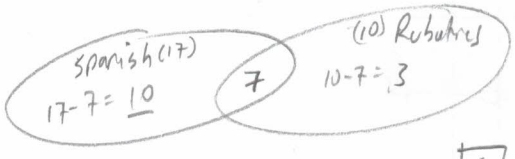
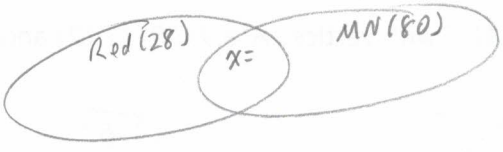
25. Penelope's school offers two clubs: a before-school Spanish Club and an after-school Robotics Club. 17 students in her class joined Robotics Club and 10 students in her class joined Spanish Club, including 7 students who joined both. Of the 26 students in Penelope's class, how many aren't a part of either club?



26 − (10 + 7 + 3) = **6**

26. During a road trip, Absame saw 113 cars go by, 28 of which were red. 80 cars had Minnesota license plates, and 22 of the cars from outside Minnesota weren't red. How many red cars with Minnesota license plates did Absame see?



] = 113 − 22 = 91

22

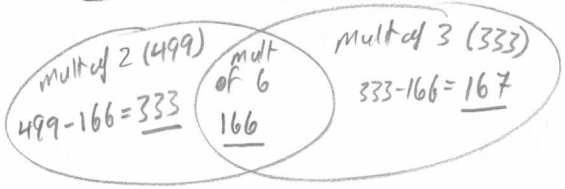
Red + MN − x = 91

28 + 80 − x = 91

x = 108 − 91 = **17**

1 to 999

27. How many positive integers less than 1000 are multiples of 2 or 3?



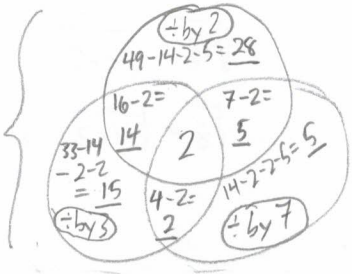
Total # of positive integers that are multiples of 2 or 3:

333 + 166 + 167 = **666**

28. How many positive integers less than 100 aren't divisible by either 2, 3, or 7?

Total is
28 + 14 + 2 + 5
+ 15 + 2 + 5
= 71

∴ # positive integers not divisible by 2, 3, or 7 is 99 − 71 = **28**



Work from "inside out"

- Divisible by 2, 3, 7 means divisible by 2·3·7 = 42: 42, 84 (2)
- Divisible by 2, 3 means divisible by 6: 6, 12, 18, ..., 96 (16)
- Divisible by 2, 7 means divisible by 14: 14, 28, 42, ..., 98 (7)
- Divisible by 3, 7 means divisible by 21: 21, 42, ..., 84 (4)
- Divisible by 2: 2, 4, 6, ..., 98 (49)
- Divisible by 3: 3, 6, 9, ..., 99 (33)
- Divisible by 7: 7, 14, 21, ..., 98 (14)