

Meet 4 Practice Problems

even exp \Rightarrow ends in 1
 \uparrow \rightarrow odd exp \Rightarrow ends in 9

W 1/6/2021
 \downarrow

1. What is the units digit of $9^4 + 9^7$?

$9^1 = 9$ 9 odd exp
 $9^2 = 81$ 1 even exp
 $9^3 = 729$ 9 odd
 $9^4 = 6561$ 1 ... even

$1 + 9 = 10$
 ends in a zero
 \therefore units digit is 0

2. Three consecutive numbers are multiplied then this number is divided by four, leaving a non-zero remainder. What is the remainder?

① Pick 3 consecutive (small) numbers, multiply them, divide by 4, check remainder.
 Keep trying until remainder is not zero.

$\frac{2 \cdot 3 \cdot 4}{4}$ $\frac{3 \cdot 4 \cdot 5}{4}$ $\frac{4 \cdot 5 \cdot 6}{4}$ $\frac{5 \cdot 6 \cdot 7}{4} = \frac{210}{4} \Rightarrow \text{rem } 2$ $\frac{1 \cdot 2 \cdot 3}{4} = \frac{6}{4} \Rightarrow \text{rem } 2$

3. Exactly one of the numbers below is a perfect square. Which one?

- 1,048,578
- 1,185,921
- 2,137,443
- ? \rightarrow - 3,771,362
- 8,156,737

$1^2 = 1 \checkmark$
 $2^2 = 4 \checkmark$
 $3^2 = 9 \checkmark$
 $4^2 \Rightarrow 6 \checkmark$
 $5^2 \Rightarrow 5 \checkmark$
 $6^2 \Rightarrow 6 \checkmark$
 $7^2 \Rightarrow 9 \checkmark$
 $8^2 \Rightarrow 4 \checkmark$
 $9^2 \Rightarrow 1 \checkmark$
 $10^2 \Rightarrow 0 \checkmark$

All perfect squares end in either 0, 1, 4, 5, 6, 9

$(12)^2 = 144$
 \uparrow
 2^2 ones digit
 $(18)^2 = 324$
 \uparrow
 8^2 ones digit

512
 \uparrow ones digit
 \uparrow tens digit

4. What is the units digit of 2^{3^4} ?

Note $2^{3^4} = 2^{(3^4)}$

$3^4 = 81$
 one more than 80, which is a multiple of 4

$2^1 = 2$ $2^2 = 4$ $2^3 = 8$ $2^4 = 16$
 $2^5 = 32$ $2^6 = 64$ $2^7 = 128$ $2^8 = 256$

\therefore one's digit is 2

5. What is the remainder when 2018^2 is divided by 13?

$2018^2 = 2018 \cdot 2018 = (2015+3)(2015+3)$ $13 \overline{) 2018} \text{ rem of } 3$

$= \frac{2015^2}{13} + \frac{2 \cdot 2015 \cdot 3}{13} + \frac{3^2}{13} \rightarrow \frac{9}{13}$

remainder is

197

$13 \overline{) 2018}$
 $\underline{155}$
 71
 $\underline{65}$
 68
 $\underline{65}$
 3

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6. What is the greatest integer value of s for which $6s - 5 > 9s + 19$?

$$\begin{array}{r}
 6s - 5 > 9s + 19 \\
 \underline{-6s} \quad \underline{-6s} \\
 -5 > 3s + 19 \\
 \end{array}
 \rightarrow
 \begin{array}{r}
 -5 > 3s + 19 \\
 \underline{-19} \quad \underline{-19} \\
 -24 > 3s \\
 \end{array}
 \rightarrow
 \begin{array}{r}
 -24 > 3s \\
 \underline{-3} \quad \underline{-3} \\
 -8 > s \\
 \end{array}
 \Rightarrow \boxed{-9} \text{ is the greatest integer}$$

7. For how many integers n are both $2n - 7$ and $50 - 3n$ positive?

$$\begin{array}{l}
 2n - 7 > 0 \\
 2n > 7 \\
 n > \frac{7}{2} = 3\frac{1}{2} \\
 \end{array}
 \quad
 \begin{array}{l}
 50 - 3n > 0 \\
 50 > 3n \\
 \frac{50}{3} > n \text{ or } 16\frac{2}{3} > n \\
 \end{array}$$

$\leftarrow \begin{array}{c} 3\frac{1}{2} \qquad 16\frac{2}{3} \\ \hline 3 \quad 4 \qquad 16 \quad 17 \end{array} \rightarrow$
 $\boxed{13}$ integers

F 1/8/2021

8. Jasmine has designed the perfect math puzzle which she wants to sell. Galois Games offers to manufacture her puzzles for \$3.50 each and charges a \$500 setup fee, while Noether Novelties will charge only \$2.30 per puzzle but requires a \$750 setup fee. What is the minimum number of puzzles for which Noether Novelties offers the lower total manufacturing cost?

Noether < Galois

Let $n = \#$ of puzzles

$$\begin{array}{r}
 2.30n + 750 < 3.50n + 500 \\
 \underline{-2.30n} \quad \underline{-2.30n} \\
 750 < 1.20n + 500 \\
 \end{array}
 \rightarrow
 \begin{array}{r}
 750 < 1.20n + 500 \\
 \underline{-500} \quad \underline{-500} \\
 250 < 1.20n \\
 \underline{1.20} \quad \underline{1.20} \\
 208.\bar{3} < n \Rightarrow n \geq \boxed{209 \text{ puzzles}}
 \end{array}$$

$\frac{25}{1.2} = \frac{2500}{12}$

9. What is the sum of all of the integer solutions to the compound inequality

$$\begin{array}{l}
 -x \leq 3 \\
 -3 \leq x \\
 \end{array}
 \rightarrow
 \begin{array}{l}
 -x \leq 3 \text{ and } 3 < 10 - x \\
 x \geq -3 \quad \frac{x-3}{x-3} \quad \frac{-3+x}{-3+x} \\
 x < 7 \\
 \end{array}$$

$-3 \leq x < 7$

$2 < x < y < z < 10$
 $2 < x < 4$ X
 Answer: $-3 + (-2) + (-1) + \dots + 6 = 4 + 5 + 6 = \boxed{15}$

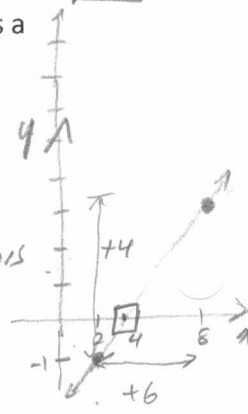
"x is strictly less than 7"

10. What is the x-intercept of the line through (2, -1) and (8, 3)? Express your answer as a common fraction.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{3 - (-1)}{8 - 2} = \frac{4}{6} = \frac{2}{3} = \frac{1}{1.5}$$

usually don't write decimals in

If y increases by 1 (to get to x -axis) from (2, -1), then x increases by 1.5, so x -intercept is $2 + 1.5 = 3.5$



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11. The cost (in dollars) of purchasing custom shirts is given by the equation

Total Cost = \$13.00 + \$7.50 × (Number of Shirts Purchased)

Let n = 5

A math club decides to purchase custom shirts to wear at competitions. Each member contributes \$8.00 and they have the exact amount needed to buy everyone a shirt. How many members does the math club have?

Total cost = (8 / t-shirt) (# t-shirts) = 8n

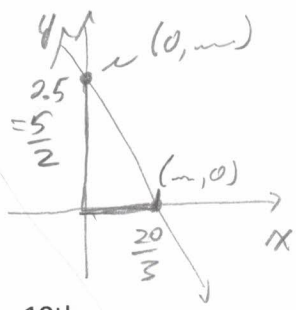
8n = 13 + 7.5n → n = 13 / 0.5 = 26 t-shirts / people (members)

12. Determine the area of the triangle formed by the x- and y- axes and the line 3x + 8y = 20

20. Express your answer as a common fraction.

To find x- and y- intercepts (and not slope)

3x + 8y = 20 → y-int is 20/8 = 2.5 (because x=0) → Area = 1/2 bh = 1/2 * 20/3 * 5/2 = 25/3 = Area



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13. The 7th term in an arithmetic sequence is 11 and its 14th term is 32. What is the 10th term in this sequence?

To go from the 7th term to the 14th term, we go from 11 to 32. That is a difference of 32-11=21 in 14-7 terms, or a common difference of 21/7=3. So the 10th term is the 7th term plus 10-7=3 common differences. ∴ the 10th term is 11 + 3(3) = 11 + 9 = 20

14. Following an outbreak of bird flu last year, the number of new cases each month has been 2/3 as many as the previous month. If there were 48 new cases of bird flu this month, how many were there three months ago?

this month: 48 × 3/2 = last month: 72 × 3/2 = two months ago: 108 × 3/2 = three months ago: 162

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15. How many non-equilateral triangles with integer sides at most 7 units long have side lengths which form an arithmetic sequence? Triangles are considered the same if their three side lengths are the same (e.g. a triangle with side lengths 2, 3, and 4 is the same as one with side lengths 4, 3, and 2).

Common difference of 1: 1-2-3, 2-3-4, 3-4-5, 4-5-6, 5-6-7

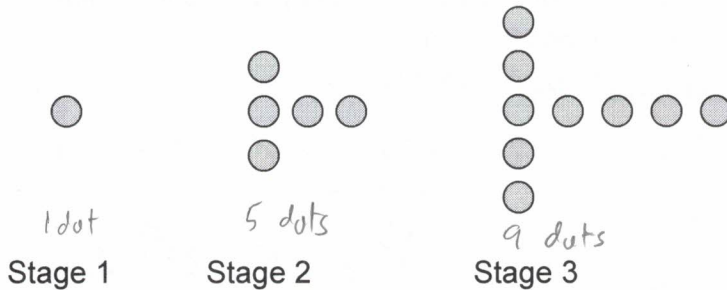
2: 1-3-5, 2-4-6, 3-5-7

3: 1-4-7

5 triangles

4 of these are not triangles because the two shortest sides must add to be greater than the longest side

16. If the pattern continues, How many dots would be drawn in stage 11?



1, 5, 9 is an arithmetic sequence with common difference 4

Stage:	1	2	3	4	5	6	7	8	9	10	11
#dots:	1	5	9	13	17	21	25	29	33	37	41

Or: $a_{11} = a_1 + (n-1)d = 1 + (11-1)(4) = 1 + 40 = 41$

where $a_1 =$ the first term $d =$ common difference = 4
 $a_{11} =$ the eleventh term

17. A right triangle has hypotenuse $\sqrt{65}$ inches and a leg of length 4 inches. What is the area of this triangle?

$$x^2 + 4^2 = (\sqrt{65})^2$$

$$x^2 + 16 = 65$$

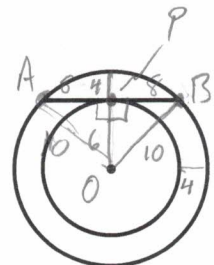
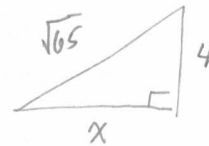
$$x^2 = 49$$

$$x = 7 > 0$$

$$\text{Area} = \frac{1}{2} \text{ base} \cdot \text{height}$$

$$= \frac{1}{2} (7)(4)$$

$$= 14$$



18. Chord AB lies on a circle of radius 10 cm and is tangent to a smaller circle of radius 6 cm as shown. Both circles share the same center, O. What is the length |AB|?

$PB = 8, PA = 8, \text{ so } AB = 16 \text{ cm}$

See figure. P is point where AB is tangent to smaller circle
 $PO = 6, OA = OB = 10$
so $PB = PA = 8$ (6-8-10 Δ)

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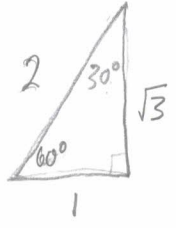
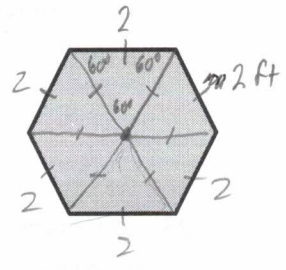


19. A kiddie pool is in the shape of a regular hexagon with sides 2 feet long. How many cubic feet of water is needed to fill the pool to a depth of 8 inches? Express your answer in simplest radical form.

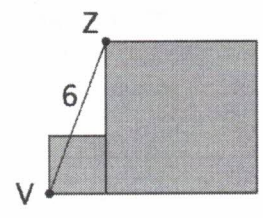
Draw the segments from the center to each vertex

$A = 6\sqrt{3}$ depth: $(8 \text{ in}) \left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = \frac{2}{3} \text{ ft}$

Volume = $A \cdot d = (6\sqrt{3}) \left(\frac{2}{3}\right) = 4\sqrt{3} \text{ ft}^3$



20. The length of segment VZ is 6 units, as shown. Find the combined area, in square units, of the two shaded squares.

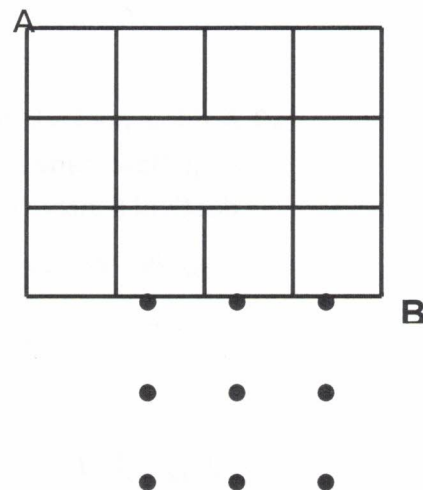


21. In how many ways can each of the numbers 1 through 6 be arranged in the blanks below so that numbers increase along each row and each number on the top row is less than the number below it?

— — —

— — —

22. How many paths are there from A to B along the network shown moving only to the right or down (but never to the left or up)?



23. How many right triangles can be formed using three points from the regular 3x3 grid shown?

24. What is the arithmetic mean (i.e. average) of the numbers 7, 77, 777, 7777, 77777, 777777, and 7777777?

25. Aung scored an average of 12.5 points per game over the first 10 games of her team's basketball season. How many points must she average over the next five games to bring her season average up to 15 points per game?

26. A set of seven positive integers has a mean of 4, a median of 4, and a unique mode of 5. What is the range of these seven numbers?

27. The arithmetic mean of the numbers in the sequence below is 180:

$$x, x + 3, x + 6, x + 9, \dots, x + 300$$