

Base 10:

$$\underline{\underline{345}} = 3 \times \underline{\underline{10}}^2 + 4 \times \underline{\underline{10}}^1 + 5 \times \underline{\underline{10}}^0$$

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W

$$1001_2 \times 13_8$$

MNJHML Meet 5 Practice Problems

W 1/20/2021
↓

1. In base 10, what is the value of $1001_2 \times 13_8$?

$$1001_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 1 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 = 8 + 1 = 9 = 9_{10}$$

$$13_8 = 1 \times 8^1 + 3 \times 8^0 = 1 \times 8 + 3 \times 1 = 8 + 3 = 11$$

$$1001_2 \times 13_8 = 9 \cdot 11 = \boxed{99} = 99_{10}$$

2. The base 9 number 62_9 is equal to base b number 132_b . What is b ?

$$62_9 = 132_b$$

$$6 \times 9 + 2 \times 1 = 1 \times b^2 + 3 \times b + 2 \times 1$$

$$56 = b^2 + 3b + 2$$

$$54 = b^2 + 3b$$

$$54 = b(b+3)$$

$\therefore b = 6$ by guess and check

3. In base 9, the fraction $\frac{1}{7}$ is represented as the repeating decimal $0.\underline{a}b\underline{c}abcabc\dots_9$, where a , b , and c are digits between 0 and 8. What is the value of c ?

$$\begin{array}{r} 0.125125\dots \\ 7 \overline{) 1.000} \\ \underline{7} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{38} \\ 2 \end{array}$$

$$\therefore \boxed{c = 5}$$

4. The following equation is true when written in base n , but the first two digits of the number $\square\square 2$ are missing. What are the missing digits? Answer by filling in the boxes.

$$\square\square 2 - 135 = 234$$

5. Anya and Adeem collect Pokemon trading cards. Together they have 95 cards, and Anya has 50% more cards than Adeem. How many Pokemon trading cards does Adeem have?

Let $m = \#$ cards Adeem has

$n =$ Anya has

$$m + n = 95 \quad m + (1.5m) = 95$$

$$n = 1.5m$$

$$\rightarrow \frac{2.5m}{2.5} = \frac{95}{2.5}$$

$$m = 38$$

\therefore Adeem has $\boxed{38}$ cards

6. If (x, y) is a solution to the pair of equations

$$x + 4y = 43$$

$$4x + y = 57$$

what is the value of $x + y$?

$$1x + 4y = 43$$

$$4x + 1y = 57$$

$$\underline{5x + 5y = 100}$$

$$\frac{5}{5} \quad \frac{5}{5}$$

$$\rightarrow \boxed{x + y = 20}$$

$$x + 4y$$

$$^3\sqrt{8} = \pm 2$$

$$^2\sqrt{4} = \pm 2$$

$$\sqrt{4}$$

$$4^2 = 16$$

$$4^1 = 4$$

7. Determine the coordinates of the point P on line $y = 15 - 2x$ which is equidistant from points $(0, 0)$ and $(-6, 6)$ (i.e. the distance from P to $(0, 0)$ is the same as the distance from P to $(-6, 6)$).

8. Solve for x :

$$(3x + 7)(x - 9) = 3x^2 - 108$$

Express your answer as a common fraction.

$$(3x + 7)(x - 9) = 3x^2 - 108$$

$$(3x)(x) + (3x)(-9) + (7)(x) + (7)(-9) = 3x^2 - 108$$

$$3x^2 - 27x + 7x - 63 = 3x^2 - 108$$

$$x - 9 = x + (-9)$$

$$\begin{array}{r} -20x - 63 = -108 \\ \quad \quad +63 \quad +63 \\ \hline -20x \quad = -45 \\ \quad \quad -20 \quad -20 \\ \hline x = \frac{9}{4} \end{array}$$

9. Evaluate: $\frac{732^2 + 2 \times 732 \times 268 + 268^2}{387^2 - 2 \times 387 \times 137 + 137^2} = \text{expression}$

$$(a+b)^2 = (a+b)(a+b) = a \cdot a + a \cdot b + b \cdot a + b \cdot b = a^2 + 2ab + b^2 = (a+b)^2$$

$$(a-b)^2 = (a-b)(a-b) = a \cdot a + a \cdot (-b) + (-b) \cdot a + (-b) \cdot (-b) = a^2 - 2ab + b^2 = (a-b)^2$$

$$\text{expression} = \frac{(732 + 268)^2}{(387 - 137)^2} = \frac{1000^2}{250^2} = \left(\frac{1000}{250}\right)^2 = 4^2 = 16$$

10. Find the largest possible value of r such that $(r - 9)(r + 9) = 8r + 47$.

$$(a+b)(a-b) = a^2 - b^2$$

Know this

$$r^2 - 9^2 = 8r + 47 \quad \text{guess \& check}$$

$$r^2 - 8r = 81 + 47$$

$$r(r - 8) = 128 \quad r = 16 > 0$$

11. Find the sum of the solutions to the equation $x^2 - 6x + 9 = 0$

$$x^2 - 6x + 9 = 0$$

$$(x - 3)^2 = 0$$

$$x = 3 \text{ is a solution, so sum} = 3$$

$$a^2 - 2ab + b^2 = 0$$

$$\text{where } a = x$$

$$b = 3$$

Better (and strictly more correct)

$$(x - 3)(x - 3) = 0$$

so there are two solutions: 3 and 3

$$3 + 3 = 6$$

$$\frac{4}{12} = \frac{1}{3}$$

$$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$$

12. Evaluate $\sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}} = x$

$$\begin{aligned} \sqrt{12+x} &= x \\ (\sqrt{12+x})^2 &= (x)^2 \\ 12+x &= x^2 \end{aligned} \Rightarrow \begin{aligned} x^2 - x &= 12 \\ x(x-1) &= 12 \\ x &= 4 \Rightarrow \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}} = 4 \end{aligned}$$

13. In the figure shown, XY and AB are parallel. If |AB| = 10, |BY| = 5, and |CY| = 3, determine |XY|. Express your answer as a common fraction.

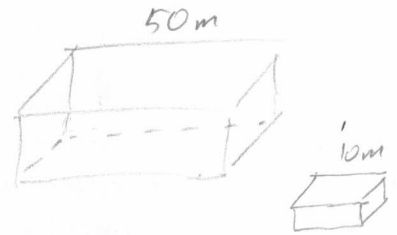
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14. An Olympic sized swimming pool is 50m long and filled with 2,500,000 liters of water.

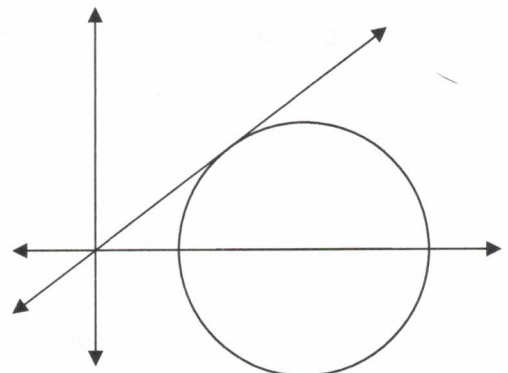
Putri's backyard pool has the exact same proportions but is only 10m long. How many liters of water are needed to fill Putri's pool?

If length is $\frac{1}{5}$ of Olympic pool,
then volume is $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$ of Olympic pool

$$V = \frac{2,500,000}{125} = 20,000 \text{ l}$$



15. Determine the equation of the line shown which passes through the origin and Quadrants I and III and is tangent to the circle centered at (5, 0) with radius 3.



16. A sock drawer contains 10 black socks, 6 white socks, and 4 blue socks. Hamza reaches in and randomly pulls out two socks. What is the probability that both are black? Express your answer as a common fraction.

$$P(1 \text{ black sock}) = \frac{10}{20}$$

$$P(2 \text{nd black sock}) = \frac{9}{19}$$

10K 6W 4B
Black White Blue

20

$$P(1 \text{st black sock}) = \frac{10}{20} \cdot \frac{9}{19} = \frac{90}{380} = \frac{9}{38}$$

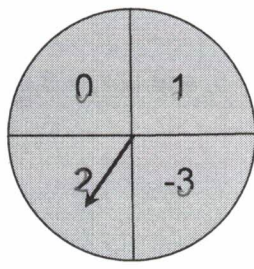
F 1/22/2021

$\frac{400}{2}$
 $\frac{400}{2}$
 $\frac{4 \times 100}{2} = 200$

F 1/22/2021

W 1/27/2021

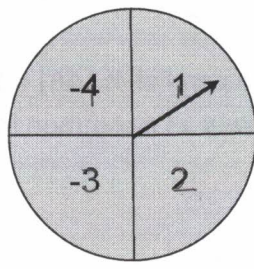
17. The two spinners below are divided into four regions each with an equal likelihood of being selected. Both spinners are spun and the two numbers produced are multiplied. What is the probability that this product is positive?



$$P(\text{pos}) = \frac{2}{4} = \frac{1}{2}$$

$$P(2 \text{ positives}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(\text{neg}) = \frac{1}{4}$$



$$P(\text{pos}) = \frac{2}{4} = \frac{1}{2}$$

$$P(\text{neg}) = \frac{2}{4} = \frac{1}{2}$$

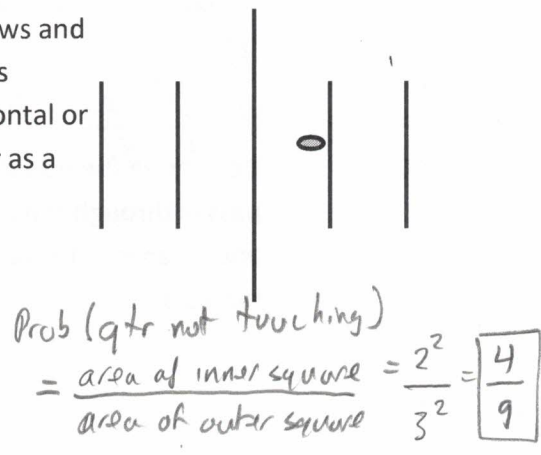
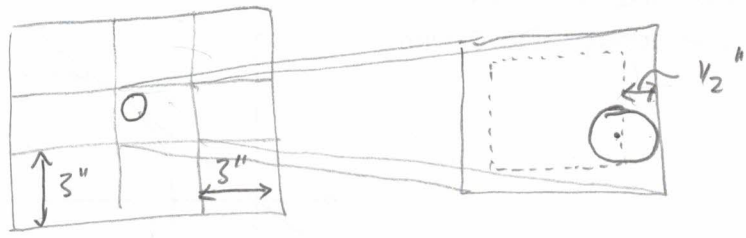
$$P(2 \text{ negs}) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(2 \text{ pos}) + P(2 \text{ neg})$$

$$= \frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8}$$

$$= \boxed{\frac{3}{8}} = P(2 \text{ \#s that multiply to a pos)}$$

18. A quarter of diameter 1" is tossed onto a square grid with rows and columns exactly 3" apart. What is the probability that it lands completely within one square, not touching any of the horizontal or vertical lines, as in the example shown? Express your answer as a common fraction.

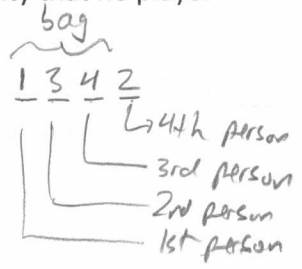


$$\begin{aligned} \text{Prob (qtr not touching)} &= \frac{\text{area of inner square}}{\text{area of outer square}} = \frac{2^2}{3^2} = \boxed{\frac{4}{9}} \end{aligned}$$

19. Four players on the soccer team left their athletic bags on the sideline during practice. If each one grabs a bag at random when they leave, what is the probability that no player gets the correct bag? Express your answer as a common fraction.

$$\# \text{ ways total} = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

- 2 1 4 3, 2 3 4 1, 2 4 1 3, 3 1 4 2, 3 4 1 2, 3 4 2 1,
- 4 1 2 3, 4 3 1 2, 4 3 2 1



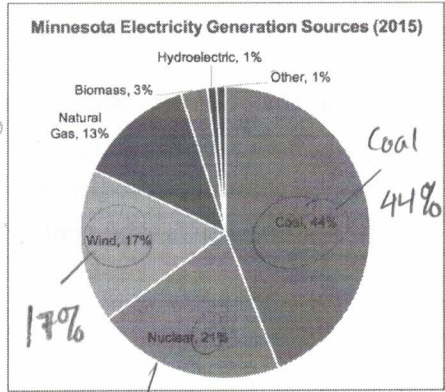
$$P(\text{no one gets their right bag}) = \frac{9}{24} = \boxed{\frac{3}{8}}$$

Probability can be written as $\frac{p}{q}$. Write $p+q$.
 $3+8 = \boxed{11}$ online

W 1/27/2021

coal
/ natural gas

20. Coal and Natural Gas are considered fossil fuels. Based on the chart given, what percentage of Minnesota's 2015 electricity was generated from sources other than fossil fuels?



$$P(\text{fossil fuels}) = P(\text{coal}) + P(\text{Nat Gas})$$

$$= 0.44 + 0.13 = 0.57$$

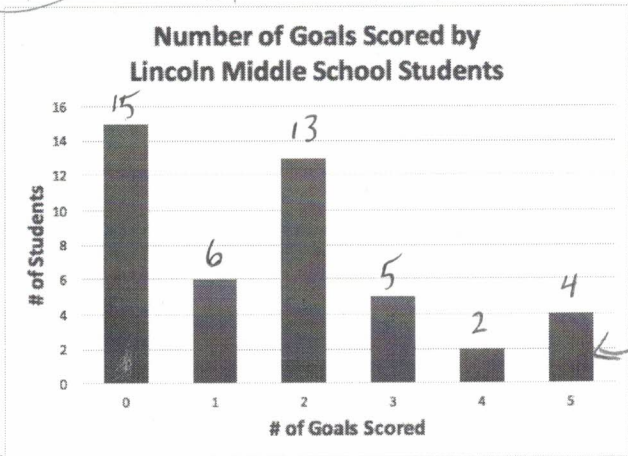
Wind

$$P(\text{not fossil fuels}) = 1 - P(\text{fossil fuels})$$

$$= 1 - 0.57$$

$$= \boxed{0.43}$$

21. 45 students at Lincoln Middle School played soccer last season. Based on the frequency histogram shown, by how many goals is the median number of goals scored per student greater than the mean number of goals scored per student? Express your answer as a



a "bin"

common fraction.

$$\text{Total \# goals} = 15 \times 0 + 6 \times 1 + 13 \times 2 + 5 \times 3 + 2 \times 4 + 4 \times 5$$

$$= 0 + 6 + 26 + 15 + 8 + 20$$

$$= 75 \text{ goals}$$

$$\text{Avg \# goals / player} = \frac{75}{45} = \frac{5}{3} \text{ goals / student}$$

Median is 2 goals (23rd highest scorer's # goals)

$$\frac{45}{2} = 22.5$$

$$\text{Median} - \text{Mean} = 2 - \frac{5}{3} = \frac{6}{3} - \frac{5}{3} = \boxed{\frac{1}{3}}$$

X

22. Lake County, Minnesota has a population of 10,800, broken into age categories as indicated in the table shown (right).

Logan wants to present this information in a pie chart. How many degrees should he measure for the central angle of the "Age 10-19" sector?

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