MSHSML 2016-17 Meet 2, Event A, #1-#4

1. Let f = # floops
b = # blorps
$2f + 3b = 6f + 1b$ Solve for b
$2b = 4f$
$b = 2f$ = **2 floops**

$1 < 4 > 2$

2. How many integers n satisfy the inequalities $2n < 7n-5 \leq 6n$

Inequality 1: $2n < 7n-5$, $5 < 5n$, $1 < n$

Inequality 2: $7n-5 \leq 6n$, $n \leq 5$

$n \in \{2, 3, 4, 5\}$ **4** integers

3. Alec mowed lawns for his summer job each of the past 5 summers. Each summer after the first, he mowed 5 more lawns and charged $2 more per lawn than during the previous summer. Determine how much Alec charged per lawn his first summer if he mowed 85 lawns over the course of those summers and made a total of $1800.

Let n = # lawns first summer
$n + (n+5) + (n+10) + (n+15) + (n+20) = 85$
$5n + 50 = 85 \Rightarrow 5n = 35 \Rightarrow n = 7$ In years 1-5: 7, 12, 17, 22, 27 lawns
Let r = rate charged per lawn. In years 1-5: r, $r+2$, $r+4$, $r+6$, $r+8$
amt earned $= 7r + 12(r+2) + 17(r+4) + 22(r+6) + 27(r+8) = 1800$
$= 85r + 24 + 68 + 132 + 216 = 1800$
$85r + 440 = 1800$
$85r = 1360 \Rightarrow r =$ **$16** ✓

4. $a, b \in \mathbb{Z}^+$ are called "equalish" if $a \neq b$ and $|a-b| \leq \min\{a, b\}$. Find the number of integers that are equalish to 2016 but not equalish to 1026.

2016 not equalish to $1, 2, 3, \ldots, 1008$
→ is equalish to $1009, 1010, \ldots, 4031$ but not 2016
not equalish to $4032, 4033, \ldots$

→ 1026 not equalish to $1, 2, 3, \ldots, 513$
is $514, 515, \ldots, 2051$ but not 1026
→ not $2052, 2053, \ldots$

equalish to 2016, not equalish to 1026
$1026, \underbrace{2052, 2053, \ldots, 4031}$
1 | 1979

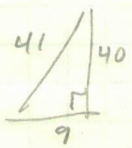
$4031 - 2052 = 1979$

1980 integers

USHSML 2016-17 Meet 2, Event B, #1, #2

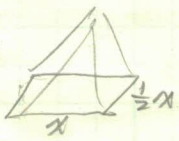
1. Determine exactly the area of a triangle whose sides have lengths 41, 9, and 40.

$\text{Area} = \frac{1}{2}(9)(40) = \boxed{180}$



$41^2 = 40^2 + 40 + 41$
1681
$= 40^2 + 9^2$ ✓

2. A pyramid has a rectangular base with a width that is one-half the length. If the height is 4 cm and the volume is 24 cm^3, find the length of the base.



$V = \frac{1}{3}$(area of base)(height)

Let x = length of rectangle

$= \frac{1}{3} \cdot x \cdot \frac{1}{2}x \cdot 4 = 24 \Rightarrow x^2 = 36 \Rightarrow x = \boxed{6 \text{ cm}}$