Stephen Mackennan
ERA Math Team
Forum AB
Tu 9 Feb 2021

Minnesota State High School Mathematics League

2018-19 State Tournament, Individual Event A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have **15** minutes for this Tournament event.

NO CALCULATORS are allowed on this event.

$n=1$ $\quad 2P = P(\ )^{8}$ $\quad t=3$

9 *yrs.* 1. A woman invests her money and notices it doubles every three years when compounded annually at r %. In how many years will she have eight times her original investment in this account?

$A = P(1+\frac{r}{n})^{nt}$

$2P = P(1+\frac{r}{100})^3 \Rightarrow 2 = (1+\frac{r}{100})^3 \Rightarrow 2^3 = ((1+\frac{r}{100})^3)^3 \Rightarrow 8P = P(1+\frac{r}{100})^{9}$

$9 = nt,\ 9 = t$ $\quad$ 9 years

$p = \frac{18}{73}$ 2. We call a date *swell* if the number of the month is a factor of the day of that month. For example, February 14 is *swell* since 2 is a factor of 14, but December 25 is not *swell* since 12 is not a factor of 25. If a day is selected randomly during the year (assuming it is not a leap year), what is the probability the day is *swell* ?

Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec

$31 + 14 + 10 + 7 + 6 + 5 + 4 + 3 + 3 + 3 + 2 + 2 = 90$

$p = \frac{90}{365} = \frac{18}{73}$

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2018-19 State Tournament, Individual Event B

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$96°$ ________

1. In *Figure 1*, $\triangle ABC$ is equilateral. Segments $\overline{AW}$, $\overline{AX}$, $\overline{AY}$, and $\overline{AZ}$ divide $\measuredangle BAC$ into five equal angles. Determine exactly the degree measure of $\measuredangle AXB$.

$a = 180° - 60° - (12° + 12°) = 180 - 84$

$a = \boxed{96°}$

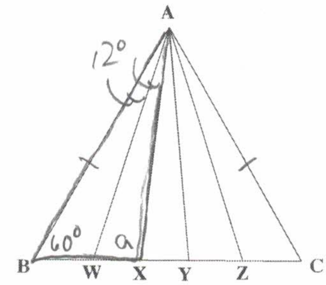


Figure 1

300 ________

2. ABC is an isosceles triangle with $\overline{AB} \cong \overline{AC}$. Point D is on $\overline{BC}$ such that the distance from D to $\overline{AB}$ is 8 and the distance from D to $\overline{AC}$ is 16. If $BC = 30$, what is the area of $\triangle ABC$?

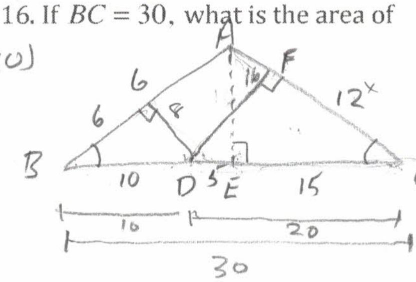
$[\triangle ABC] = \frac{1}{2}(20)(30)$

$= \boxed{300}$

$\triangle AEC \sim \triangle DFC \sim \triangle DGB$

$\frac{AE}{DF} = \frac{AC}{DC} \Rightarrow \frac{AE}{16} = \frac{AC}{20}$

$\frac{AE}{EC} = \frac{GD}{BD} \Rightarrow AE = \cancel{20}15 \cdot \frac{8}{\cancel{10}6} = 20$



Stephen Maclennan
ELA Math Team
Form AB
Tu 9 Feb 2021

Minnesota State High School Mathematics League

2019-20 State Tournament, Individual Event A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have **15** minutes for this Tournament event.

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289 1. How many numbers are in this linear sequence: 4, 11, 18, 25, ... , 2020 ?

$+7 \quad +7 \quad +7$

$\frac{2020-4}{7}+1$

$a_n = 4 + 7d, \; n = 0, 1, 2, \ldots$

$a_n = 4 + 7(d-1), \; n = 1, 2, 3, \ldots$

$= \frac{2016}{7} + 1 = 300 - 12 + 1 = \boxed{289}$

($-rs$, $r+s$) 2. Line ℓ_1 is $x + ry = r^2$ and line ℓ_2 is $x + sy = s^2$. If $r \neq s$, express the coordinates of the point of intersection of these two lines in terms of r and s. (The coordinates must be simplified as much as possible to receive credit.)

$\ell_1 - \ell_2 : ry - sy = r^2 - s^2 \Rightarrow (r-s)y = (r+s)(r-s) \Rightarrow \boxed{y = r+s}$

$\ell_1 : x + r(r+s) = r^2 \Rightarrow x + r^2 + rs = r^2 \Rightarrow \boxed{x = -rs}$

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Minnesota State High School Mathematics League

2019-20 State Tournament, Individual Event B

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$\frac{3}{4}$ ________ 1. The length of altitude $\overline{AD}$ of $\triangle ABC$ is $\frac{6}{7}$ the length of the altitude $\overline{PT}$ of $\triangle PQR$. If $BC = \frac{7}{8}QR$, determine exactly the ratio of the area of $\triangle ABC$ to the area of $\triangle PQR$.

$$\frac{[\triangle ABC]}{[\triangle PQR]} = \frac{\cancel{\tfrac{1}{2}}(BC)(AD)}{\cancel{\tfrac{1}{2}}(QR)(PT)} = \frac{\frac{7}{8}\cancel{QR}\cdot\frac{6}{7}\cancel{PT}}{\cancel{QR}\cdot\cancel{PT}} = \frac{6}{8} = \boxed{\frac{3}{4}}$$

$48°$ ________ 2. In *Figure 2*, equilateral triangle ABC is inscribed in a circle. Points D, E, F, G divide minor arc BC into five equal arcs. Secants $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at P. Determine exactly the measure of $\angle APC$.

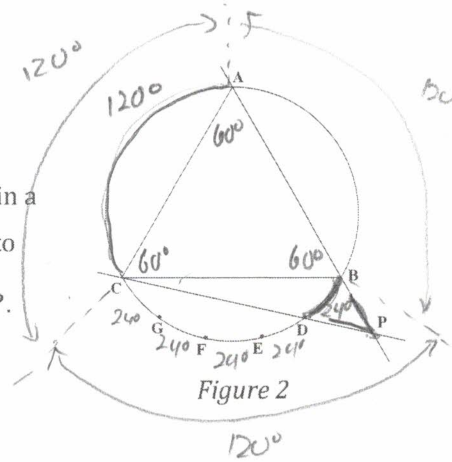

Figure 2

$m\angle APC = \frac{1}{2}(m\widehat{AC} - m\widehat{BD}) = \frac{1}{2}(120-24) = 48°$