Math Team

MSHSML Meet 6 Events AB Problem 3 Practice (2018-19 and 2019-20)

Event A

Problem #3 ("textbook with a twist"; 2 points)

$214_6 = 2\times6^2 + 1\times6^1 + 4\times6^0$

Try to solve each problem within three minutes.

3. Compute the smallest possible integer value for $b > 2$, such that $\sqrt{0.12_b}$ is a rational number in base 10. (MSHSML 2019-20 6A #3)

$$\sqrt{0.12_b} = \sqrt{1\times b^{-1} + 2\times b^{-2}} = \sqrt{\frac{1}{b} + \frac{2}{b^2}} = \sqrt{\frac{1}{b}\cdot\frac{b}{b} + \frac{2}{b^2}} = \sqrt{\frac{b}{b^2} + \frac{2}{b^2}}$$

$$= \sqrt{\frac{b+2}{b^2}} = \frac{\sqrt{b+2}}{\sqrt{b^2}} = \frac{\sqrt{b+2}}{b} = \frac{\sqrt{7+2}}{7} = \frac{3}{7} \Rightarrow b = 7$$

Event A

Problem #3 ("textbook with a twist"; 2 points)

Try to solve each problem within three minutes.

3. Determine exactly the ordered quadruple (w, x, y, z) which satisfies this system:

$$2w + x + y + z = 5$$
$$w + 2x + y + z = 10$$
$$w + x + 2y + z = 20$$
$$w + x + y + 2z = 40$$

(MSHSML 2018-19 6A #3)

$2x + 4y = 12$

$3x - 4y = 6$

$5w + 5x + 5y + 5z = 75$

$w + x + y + z = 15$ ★

$2w + x + y + z = 5$

$-(w + x + y + z = 15)$

$w = -10$

Similarly, $x = 10 - 15 = -5$

$y = 20 - 15 = 5$

$z = 40 - 15 = 25$

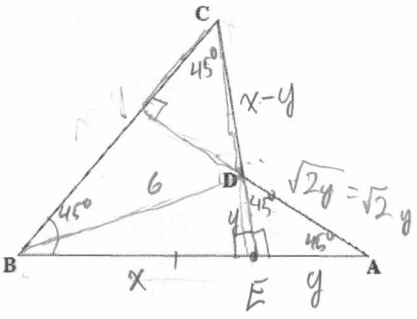
$\therefore (w, x, y, z) = (-10, -5, 5, 25)$

Event B

Problem #3 ("textbook with a twist"; 2 points)

Try to solve each problem within three minutes.

3. In the figure, concave quadrilateral $ABCD$ is concave at D. Interior angles A, B, and C are congruent and $m\angle D = 225°$. If $BD = 6$, determine exactly the area of quadrilateral $ABCD$. (Figure not drawn to scale.) (MSHSML 2019-20 6B #3)



Area of $\triangle BCE$ $= [BCE] = \frac{x^2}{2}$ $\quad [EAD] = \frac{y^2}{2}$

$x^2 + y^2 = 36$

$x - y = \sqrt{2}y$

$x = y + \sqrt{2}y = (1+\sqrt{2})y$

$(1+\sqrt{2})^2 y^2 + y^2 = 36$

$y^2(1+2\sqrt{2}+2+1) = 36$

$y^2 = \frac{36}{4+2\sqrt{2}} = \frac{18}{2+\sqrt{2}}$

$x^2 = 36 - y^2 = \frac{36(2+\sqrt{2})}{2+\sqrt{2}} - \frac{18}{2+\sqrt{2}}$

$= \frac{54+36\sqrt{2}}{2+\sqrt{2}}$

$\frac{x^2}{2} + \frac{y^2}{2} = \frac{54+36\sqrt{2}+18}{2(2+\sqrt{2})}$

$\frac{1}{2}(x^2+y^2)$

$= \frac{72+36\sqrt{2}}{2(2+\sqrt{2})} = \frac{36+18\sqrt{2}}{2+\sqrt{2}}$

$= \frac{18(2+\sqrt{2})}{2+\sqrt{2}}$

$= \boxed{18}$ ✓

Event B

Problem #3 ("textbook with a twist"; 2 points)

Try to solve each problem within three minutes.

3. Right triangle ABC has legs $\overline{AB}$ and $\overline{BC}$ of lengths 20 and 21, respectively. M is the midpoint of $\overline{AB}$ and N is the trisection point of $\overline{BC}$ closest to C. If $\overline{AN}$ and $\overline{CM}$ intersect at O and ray $\overrightarrow{BO}$ intersects $\overline{AC}$ at P, determine exactly the area of $\triangle ABP$. (MSHSML 2018-19 6B #3)