

Math Team

Meet 4 Events A and B Problems 1-2 Practice (2018-19 through 2019-20)

Event A

Problem #1 ("quickie"; 1 point)

Try to solve each problem within one minute.

1. Determine exactly the value of $\frac{9^3}{3^9}$. (MSHSML 2019-20 4A #1)

$$\frac{9^3}{3^9} = \frac{(3^2)^3}{3^9} = \frac{3^6}{3^9} = \frac{1}{3^3} = \boxed{\frac{1}{27}} = 3^{-3}$$

Not acceptable

1. If $p^2 = 2020 + q^2$ and $p = 10 + q$, compute $p + q$.
(MSHSML 2018-19 4A #1)

$$p^2 - q^2 = 2020$$
$$p - q = 10$$

$$(p+q)(p-q) = 2020$$

$$(p+q)(10) = 2020$$

$$p+q = \boxed{202}$$

Event A

Problem #2 ("textbook"; 2 points)

Try to solve each problem within two minutes.

21. Determine exactly the solution to $\frac{3x+2}{x+5} - \frac{8x+6}{3x+15} = 1$

1. (MSHSML 2019-20 4A #2)

$$\frac{3}{3} \cdot \frac{3x+2}{x+5} - \frac{8x+6}{\underbrace{3x+15}_{3(x+5)}} = \frac{9x+6}{3(x+5)} - \frac{8x+6}{3(x+5)} = \frac{9x+6-8x-6}{3(x+5)} = \frac{x}{3(x+5)} = 1$$

$$x = 3(x+5) \rightarrow 2x = -15$$

$$x = 3x + 15 \rightarrow x = \boxed{\frac{-15}{2}}$$

21. If a and b are positive real numbers and $\frac{a^2+b^2}{\frac{1}{a^2}+\frac{1}{b^2}} = 10$,

determine exactly the value of $\frac{a^3+b^3}{\frac{1}{a^3}+\frac{1}{b^3}}$. (MSHSML 2018-19 4A #2)

$$\frac{a^2+b^2}{\frac{1}{a^2}+\frac{1}{b^2}} \cdot \frac{a^2b^2}{a^2b^2} = \frac{a^2b^2(a^2+b^2)}{b^2+a^2} = a^2b^2 = 10 \quad \text{exp: 2}$$

$$ab = \sqrt{10}$$

$$\frac{a^3+b^3}{\frac{1}{a^3}+\frac{1}{b^3}} \cdot \frac{a^3b^3}{a^3b^3} = \frac{a^3b^3(a^3+b^3)}{b^3+a^3} = a^3b^3 = (a^2b^2)^{3/2} = 10^{3/2} = \sqrt{1000}$$

$$= \sqrt{100 \cdot 10} = \sqrt{100} \cdot \sqrt{10} = \boxed{10\sqrt{10}}$$

$$a^3b^3 = ab^3 = (\sqrt{10})^3 = \boxed{10\sqrt{10}}$$

Event B

Problem #2 ("textbook"; 2 points)

Try to solve each problem within two minutes.

2. One circle has a circumference of 1 and a diameter whose length is a . Another circle has an area of 1 and a diameter whose length is \sqrt{b} . Determine exactly the value of $\frac{a}{b}$.

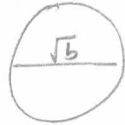
[calculator allowed] (MSHSML 2019-20 4B #2)



$$2\pi r = 1$$

$$2\pi \frac{a}{2} = 1$$

$$a = \frac{1}{\pi}$$



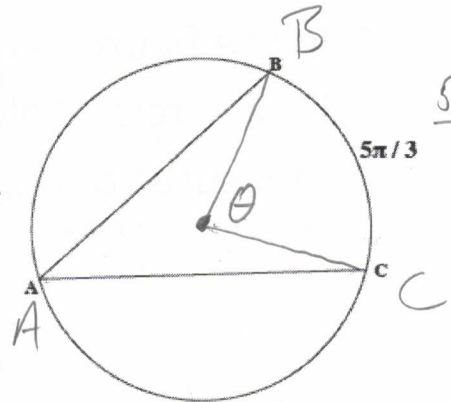
$$\pi r^2 = 1$$

$$\pi \left(\frac{\sqrt{b}}{2}\right)^2 = 1 \Rightarrow \pi \frac{b}{4} = 1 \Rightarrow b = \frac{4}{\pi}$$

$$\therefore \frac{a}{b} = \frac{\frac{1}{\pi}}{\frac{4}{\pi}} = \boxed{\frac{1}{4}}$$

2. In the figure, a circle of radius 20 contains three points A, B, and C. Two chords, \overline{AB} and \overline{AC} , are drawn. If the length of \widehat{BC} is $\frac{5\pi}{3}$, determine exactly the measure of $\angle BAC$. [calculator

allowed] (MSHSML 2018-19 4B #2)



$$\frac{5\pi}{3} = s$$

No! $s = r\theta$

$$\left(\frac{1}{2}\right)r\theta = s = \text{arc length}$$

$$\frac{1}{2} \cdot 20 \cdot \theta = \frac{5\pi}{3}$$

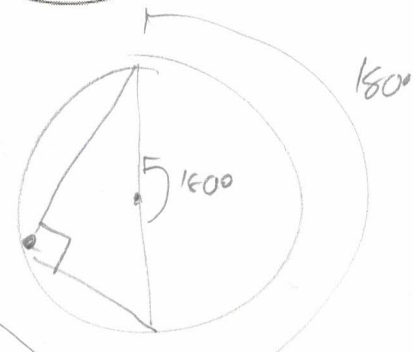
Or set up ratios:

$$\frac{5\pi/3}{40} = \frac{\theta}{20}$$

$$\frac{\pi}{12} = \frac{10\pi/3}{40} = \theta$$

$$20 \theta = \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{12} \Rightarrow m\angle BAC = \frac{1}{2}\theta = \boxed{\frac{\pi}{12}}$$



$$m\angle BAC = \boxed{\frac{\pi}{24}}$$

$$\frac{1}{2}\theta = \boxed{\frac{\pi}{24}}$$