

Math Team Notes
MSHSML Meet 4 Events C and D Problems 1-2 (2018-19 and 2019-20)

Event C

Problem #1 ("Quickie"; 1 point)

Try to solve each problem within one minute.

1. Determine exactly the value of this infinite geometric

sum: $\frac{2}{125} + \frac{4}{625} + \frac{8}{3125} + \dots$. [calculator allowed] {MSHSML 2019-20 4C #1}

$$\frac{2^1}{5^3} + \frac{2^2}{5^4} + \frac{2^3}{5^5} + \dots = \frac{1}{5^2} \left(\frac{2}{5} + \frac{2^2}{5^2} + \frac{2^3}{5^3} + \dots \right)$$

$$= \frac{1}{25} \sum_{n=1}^{\infty} \left(\frac{2}{5} \right)^n = \frac{1}{25} \left(\frac{\frac{2}{5}}{1 - \frac{2}{5}} \right) = \frac{1}{25} \cdot \frac{2/5}{3/5} = \frac{1}{25} \cdot \frac{2}{3} = \boxed{\frac{2}{75}}$$

$$\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$$

$|r| < 1$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 2$$

1. Determine exactly the value of this infinite sum: $4 + \frac{4}{3} +$

$\frac{4}{9} + \dots$. (MSHSML 2018-19 4C #1)

$$\frac{4}{1} + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \dots = 4 \left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \right)$$

$$= 4 \cdot \left[\left(\frac{1}{3} \right)^0 + \left(\frac{1}{3} \right)^1 + \left(\frac{1}{3} \right)^2 + \left(\frac{1}{3} \right)^3 + \dots \right] = 4 + 4 \left[\left(\frac{1}{3} \right)^1 + \left(\frac{1}{3} \right)^2 + \left(\frac{1}{3} \right)^3 + \dots \right]$$

$$= 4 + 4 \sum_{n=1}^{\infty} \left(\frac{1}{3} \right)^n = 4 + 4 \left(\frac{1/3}{1 - 1/3} \right) = 4 + 4 \left(\frac{1/3}{2/3} \right) = 4 + 4 \cdot \frac{1}{2} = \boxed{6}$$

Event C

Problem #2 ("Textbook"; 2 points)

Try to solve each problem within two minutes.

$a_3 = \frac{a_2}{a_1} = \frac{6}{3} = 2$ recursion equation or formula

2. $a_1 = 3, a_2 = 6$, and $a_n = \frac{a_{n-1}}{a_{n-2}}$ is a periodic sequence

with a period of 6. Determine exactly the value of

a_{2020} . [calculator allowed] (MSHSML 2019-20 4C #2)

$a_1 = 3 = a_7 = \frac{a_6}{a_5} = \frac{1/2}{1/6} = 3 = a_{13} = a_{19} = \dots$

$a_2 = 6 = a_8 = a_{14} = a_{20} = \dots$

$a_3 = \frac{a_{3-1}}{a_{3-2}} = \frac{a_2}{a_1} = \frac{6}{3} = 2$

$a_4 = \frac{a_3}{a_2} = \frac{2}{6} = \frac{1}{3}$

$a_5 = \frac{a_4}{a_3} = \frac{1/3}{2} = \frac{1}{6}$

$a_6 = \frac{a_5}{a_4} = \frac{1/6}{1/3} = \frac{1}{2}$

$a_{2020} = \frac{1}{3}$

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6) 2020

18

22

18

40

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2. What is the value of the sum $(1 + 2 - 3) + (4 + 5 - 6) + (7 + 8 - 9) + \dots + 242 - 243$? (MSHSML 2018-19 4C #2)

$= 0 + 3 + 6 + 9 + \dots + 240 = 3(1 + 2 + 3 + \dots + 80)$

$\sum_{n=1}^{80} 3n = 3 \sum_{n=1}^{80} n = 3 \cdot \frac{80 \cdot 81}{2} = 3 \cdot 3240 = 9720$

$\sum_{k=1}^n k = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

$1^2 + 2^2 + 3^2 + \dots + n^2 =$

Event D

Problem #1 ("Quickie"; 1 point)

Try to solve each problem within one minute.

1. A parabola has a minimum value of -7 and x -intercepts of -2 and 16 . What are the coordinates of its vertex? (MSHSML 2019-20 4D #1)

1. What are the coordinates of the vertex of the parabola $y = 3x^2 - 12x + 7$? (MSHSML 2018-19 4D #1)

Event D

Problem #2 ("Textbook"; 2 points)

Try to solve each problem within two minutes.

2. Determine exactly the distance between the vertices of the two parabolas determined by $y_1 = -x^2 + 2x$ and $y_2 = 2x^2 + 4x + 3$. (MSHSML 2019-20 4D #2)

2. A hyperbola has $y = \frac{5}{2}x + 24$ and $y = -\frac{5}{2}x + 4$ as its asymptotes and has a vertex at $(-4, 19)$. What are the coordinates of the other vertex? (MSHSML 2018-19 4D #2)

Th 7 Jan 2020

ERA Sol Math Team

If $10^2=100$, why is

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$9 \cdot 11 = 100 - 1^2$ and $8 \cdot 12 = 100 - 2^2$ and $7 \cdot 13 = 100 - 3^2$?

$$n^2 = n \cdot n$$

$$(n+1)(n-1) = n^2 - \cancel{n+n} - 1 = n^2 - 1$$

$$(n+2)(n-2) = n^2 - \cancel{2n+2n} + 2^2 = n^2 - 2^2 = n^2 - 4$$

$$(n+3)(n-3) = n^2 - \cancel{3n+3n} + 3^2 = n^2 - 3^2 = n^2 - 9$$

$$28 = 2^2 \cdot 7 = 1 \cdot 2^2 \cdot 7 = 1^2 \cdot 2^2 \cdot 7$$

prime
factorization

if 1 were allowed to be prime,
then integers would no longer
have a unique factorization