

Math Team
Meet 1 Events AB Problems 1-2 2016-17 Practice

Event A

Problem #1 ("quickie"; 1 point)

Try to solve each problem within one minute.

1. Express $\frac{\frac{4}{3} - \frac{3}{4}}{\frac{3}{4} + \frac{4}{3}}$ as a quotient of two relatively prime integers.

(MSHSML 2017-18 1A #1)

$$\frac{\frac{4}{3} - \frac{3}{4}}{\frac{3}{4} + \frac{4}{3}} = \frac{\frac{16-9}{12}}{\frac{9+16}{12}} = \frac{16-9}{16+9} = \frac{7}{25}$$

answer is $\frac{a}{b}$ write $a+b = 7+25 =$

32

$$5 = \frac{5}{1}$$

6

$$-\frac{3}{4} = \frac{-3}{4} \Rightarrow$$

11

2. Express $\frac{2}{3} + \frac{5}{\frac{5}{3} + \frac{5}{6}}$ as a quotient of two relatively prime integers. (MSHSML 2016-17 1A #1)

$$\frac{2}{3} + \frac{5}{\frac{5}{3} + \frac{5}{6}} = \frac{2}{3} + \frac{30}{10+5} = \frac{2}{3} + 2 = \frac{2}{3} + \frac{6}{3} = \frac{8}{3}$$

Problem #2 ("textbook"; 2 points)

Try to solve each problem within two minutes.

1. Compute $\frac{lcm(20,18)}{gcd(20,18)}$. (MSHSML 2017-18 1A #2)

LCM / GCD

$$\frac{lcm(2 \cdot 2 \cdot 5, 2 \cdot 3 \cdot 3)}{gcd(2, 3)} = \frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}{2} = 90$$

2. Find the base-nine number that is equivalent to 245_6 .

(MSHSML 2016-17 1A #2)

$$\begin{aligned} 245_6 &= 2 \times 6^2 + 4 \times 6^1 + 5 \times 6^0 = 72 + 24 + 5 = 101 = \underline{\quad} \times 9^2 + \underline{\quad} \times 9^1 + \underline{\quad} \times 9^0 \\ &= \underline{1} \times 81 + \underline{2} \times 9 + \underline{2} \times 1 \\ &= \underline{122}_9 \end{aligned}$$

$$245_{10} = 2 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$$

Event B

Problem #1 ("quickie"; 1 point)

Try to solve each problem within one minute.

1. Right triangle $\triangle ABC$ has legs of lengths $3\sqrt{2}$ and $4\sqrt{2}$.

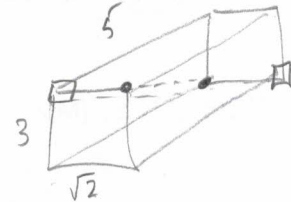
Determine exactly the length of the hypotenuse. (MSHSML 2017-18 1B #1)

$5\sqrt{2}$

(3,4,5) is a Pythagorean triple,

2. A rectangular box has faces whose side lengths are $\sqrt{2}$, 3, and 5. Find the longest diagonal of the box. (MSHSML 2016-17 1B #1)

$d = \sqrt{(\sqrt{2})^2 + 3^2 + 5^2} = \sqrt{2 + 9 + 25} = \sqrt{36} = 6$



Problem #2 ("textbook"; 2 points)

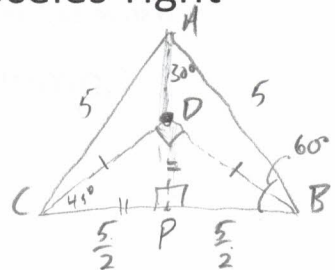
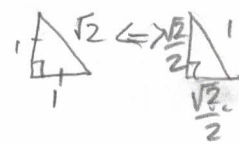
Try to solve each problem within two minutes.

1. Equilateral $\triangle ABC$ has side length of 5. Point D is in the interior of $\triangle ABC$ such that $\triangle DCB$ is an isosceles right triangle. Determine exactly AD . (MSHSML 2017-18 1B #2)

altitude of $\triangle ABC$ is $\frac{5\sqrt{3}}{2}$
 $AD = \frac{5\sqrt{3}}{2} - \frac{5}{2} = \frac{5\sqrt{3} - 5}{2}$

$DP = \frac{5}{2}$

$45-45-90$



2. $\triangle ABC$ is an isosceles triangle whose hypotenuse \overline{AC} has a length of $9\sqrt{6}$. If point D lies on \overline{BC} such that $m\angle BAD = 30^\circ$, determine exactly AD . (MSHSML 2016-17 1B #2)

$BC = \frac{9\sqrt{6}}{\sqrt{2}} = 9\sqrt{3} = AB$

18

$30-60-90$

