-Rec

Triangular Numbers

n	$T_n = \frac{1}{2}n(n+1)$
1	1
2	$1+2=3$
3	$1+2+3=6$
4	$1+2+3+4=10$
5	$1+2+3+4+5=15$
6	
7	
8	
9	
10	$1+2+\cdots+10=\boxed{55}$

Square Numbers

n	$S_n = n^2$	
1	1	$= 0+1$ special case
2	$1+3=4$	$= 1+3 = T_1+T_2$ ✓
3	$1+3+5=9$	$= 3+6 = T_2+T_3$ ✓
4	$1+3+5+7=16$	$= 6+10 = T_3+T_4$ ✓
5	$1+3+5+7+9=25$	$= 10+15 = T_4+T_5$ ✓
6		

Show that 2 triangular numbers sum to a square number

$$T_n + T_{n-1} = \frac{1}{2}n(n+1) + \frac{1}{2}(n-1)n$$

$$= \frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2}n^2 - \frac{1}{2}n$$

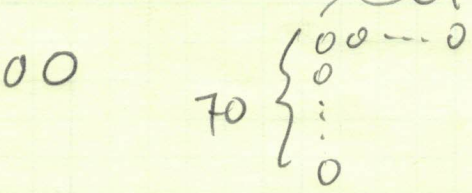
$$= n^2 = S_n$$

Know squares up to $32^2 = 1024$

Puzzle #1

What is the smallest number of balls that can first be arranged as a square
o o o
o o o
o o o
and then arranged in a square pyramid? 70

One solution: 4900



Which pyramidal number is 4900?

Puzzle #2

You have two same-sized tetrahedrons and want to combine them into a single large tetrahedron. What is the minimum number of balls/oranges for which you can do this? Ans: 20 balls/oranges

Tetrahedal Numbers

n	$TH_n = \frac{1}{6} n(n+1)(n+2)$
1	1
2	4
3	10
4	20
5	35
6	56
7	84
8	⋮

Square Pyrimidal Numbers

n	$SP_n = \frac{1}{6} n(n+1)(2n+1)$	
1	1	
2	$1+4=5$	$= 4+1$
3	$1+4+9=14$	$= 10+4$
4	$1+4+9+16=30$	$= 20+10$
5	$1+4+9+16+25=55$	$= 35+20$
6		
⋮		

Sphere packing

3rd row = 2 choices

- hexagonal close-packing
- cubic close-packing

Each has the same density of

$\frac{\pi}{\sqrt{18}} \approx 0.74 = 74\%$ believed to be the best possible

Dropping balls randomly gives a density of 0.59 to 0.63

Puzzle #3

A rectangular box is 10 in × 10 in × 5 in

What is the largest number of spheres 1 inch in diameter that can be packed in the box?

Hint: balls may not be stacked on top of each other