

Sprouts

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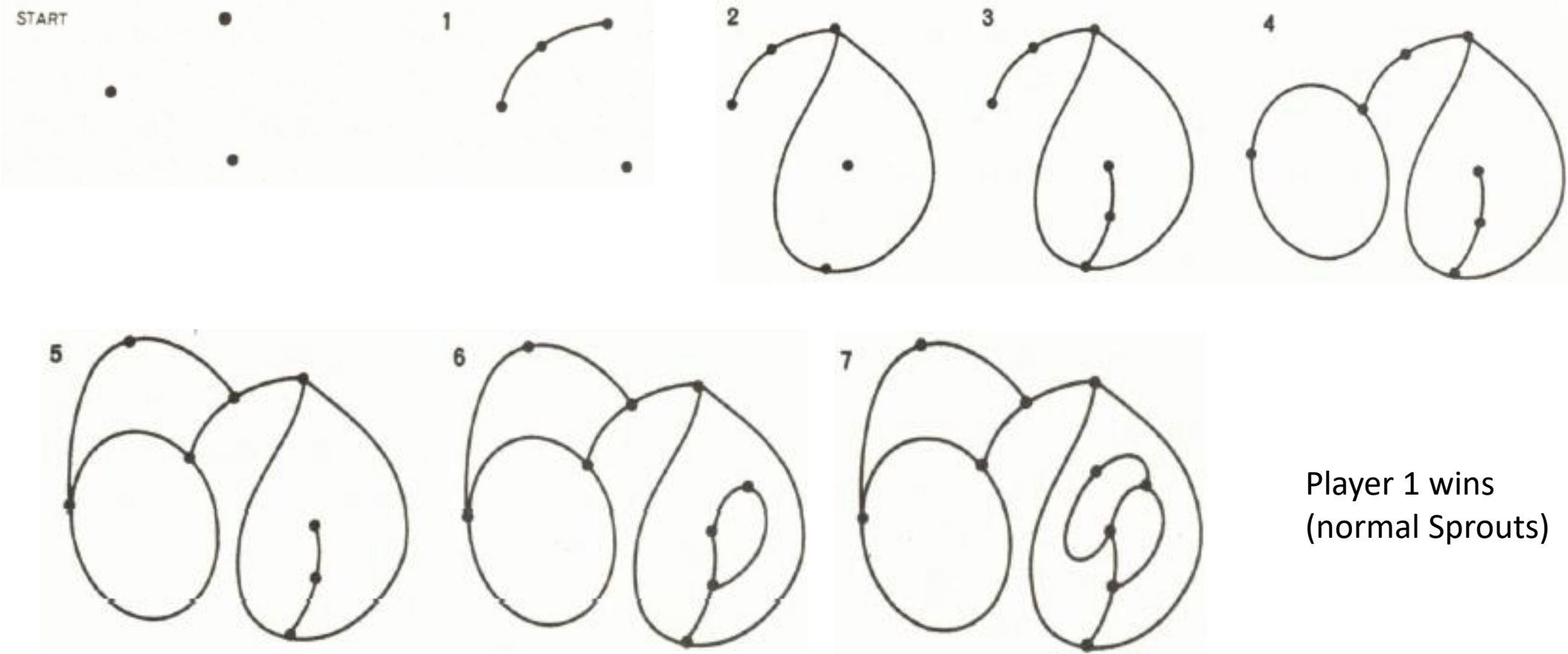
Math Team Symposium

9 October 2020

Sprouts Rules

1. Draw n spots on a piece of paper. When first learning, try $n = 3$.
2. The first player draws a curve that joins one spot with another and places a new spot anywhere along the curve, such that:
 - The curve may have any shape but it must not cross itself, cross a previously drawn curve, or pass through a previously made spot.
 - No spot may have more than three curves emanating from it.
3. Players take turns drawing curves and placing spots.
4. In normal Sprouts, the winner is the last person able to play. In reverse Sprouts, the winner is the first person unable to play.

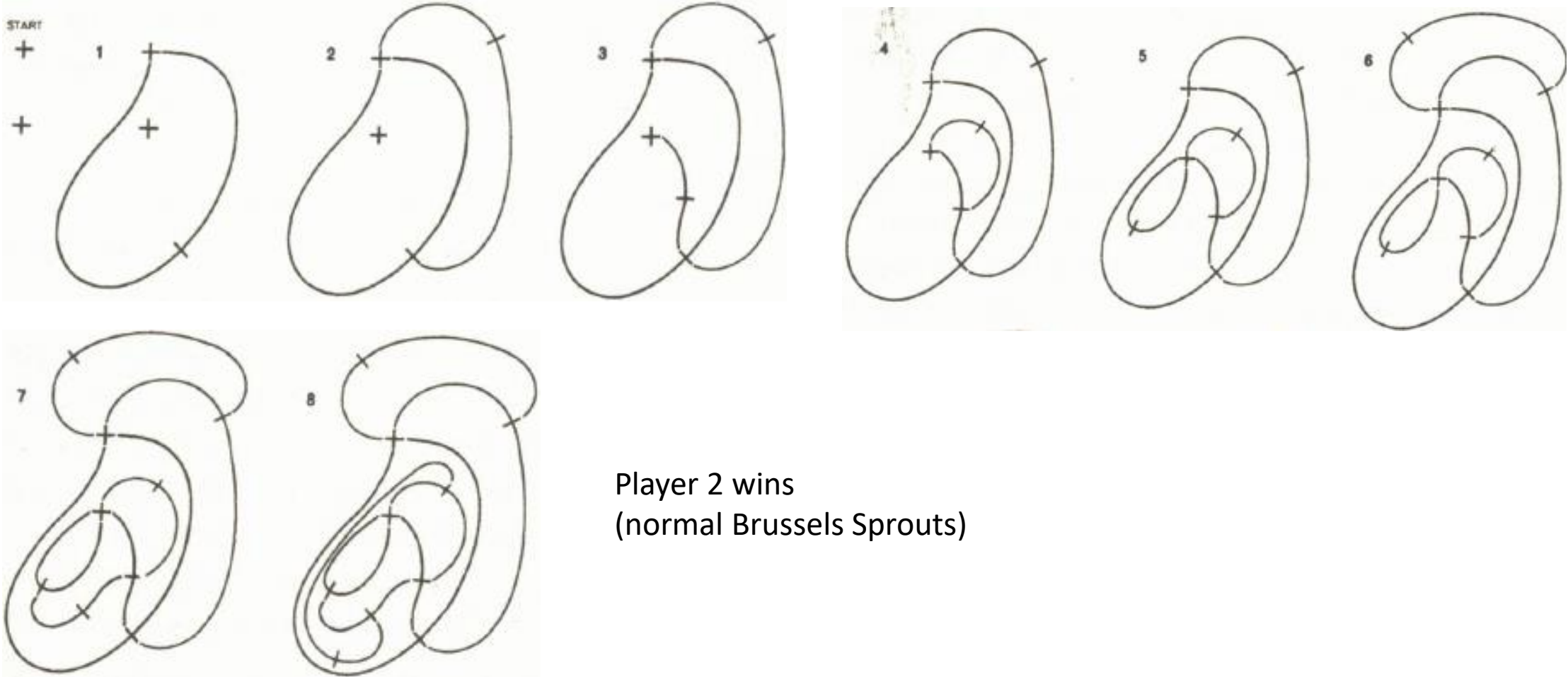
Three-Spot Sprouts Example



Brussels Sprouts Rules

1. Begin with n crosses instead of spots.
2. A move consists of extending any arm of any cross into a curve that ends at the free arm of any other cross or the same cross; then a crossbar is drawn anywhere along the curve to create a new cross. Two arms of the new cross will, of course, be dead, since no arm may be used twice.
3. As in Sprouts, no curve may cross itself or cross a previously drawn curve, nor may it go through a previously made cross.
4. As in Sprouts, the winner of the normal game is the last person to play, and the winner of the reverse game is the first person who cannot play.

Two-Cross Brussels Sprouts Example



Variations

1. In Sprouts, when placing a spot, indicate with a tiny arrow to one side of the curve, and allow new curves to be drawn only to the arrow's point.
2. In Sprouts, a player, on each turn, has a choice of adding one, two, or no spots to the curve drawn.
3. In Brussels Sprouts, replace crosses by "stars" – m crossbars crossing at the same point (like an asterisk).

Aftermath (Sprouts)

1. With $n = 1$ initial spots, how many moves can there be?
 - With $n = 2$? With $n = 3$?
2. With n initial spots, what is the maximum number of moves, in terms of n ?
 - What is the minimum number of moves in terms of n ?
3. With $n = 1$, which player (Player 1 or Player 2) always wins?
 - With $n = 2$? With $n = 3$?
4. What rule modifications might you make to make the game even more interesting?

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History

- The game of Sprouts was invented on the afternoon of 21 February 1947 by John Horton Conway, a teacher of mathematics at Sidney Sussex College, Cambridge, England, and Michael Stewart Paterson, a graduate student working at Cambridge on abstract computer programming theory. The inventors mutually agreed Mr. Paterson is to be given $\frac{3}{5}$ credit for inventing the game, and Professor Conway $\frac{2}{5}$ credit, because it was Mr. Paterson's idea to put a new dot on the curve.
- Professor Conway later invented the game of Brussels Sprouts, more as a joke. Work through the Aftermath questions to get the joke.

References

- Gardner, M. (2001). Sprouts and Brussels sprouts. *The colossal book of mathematics* (pp. 485-492). W. W. Norton & Company.